NATURAL

PHILOSOPHY

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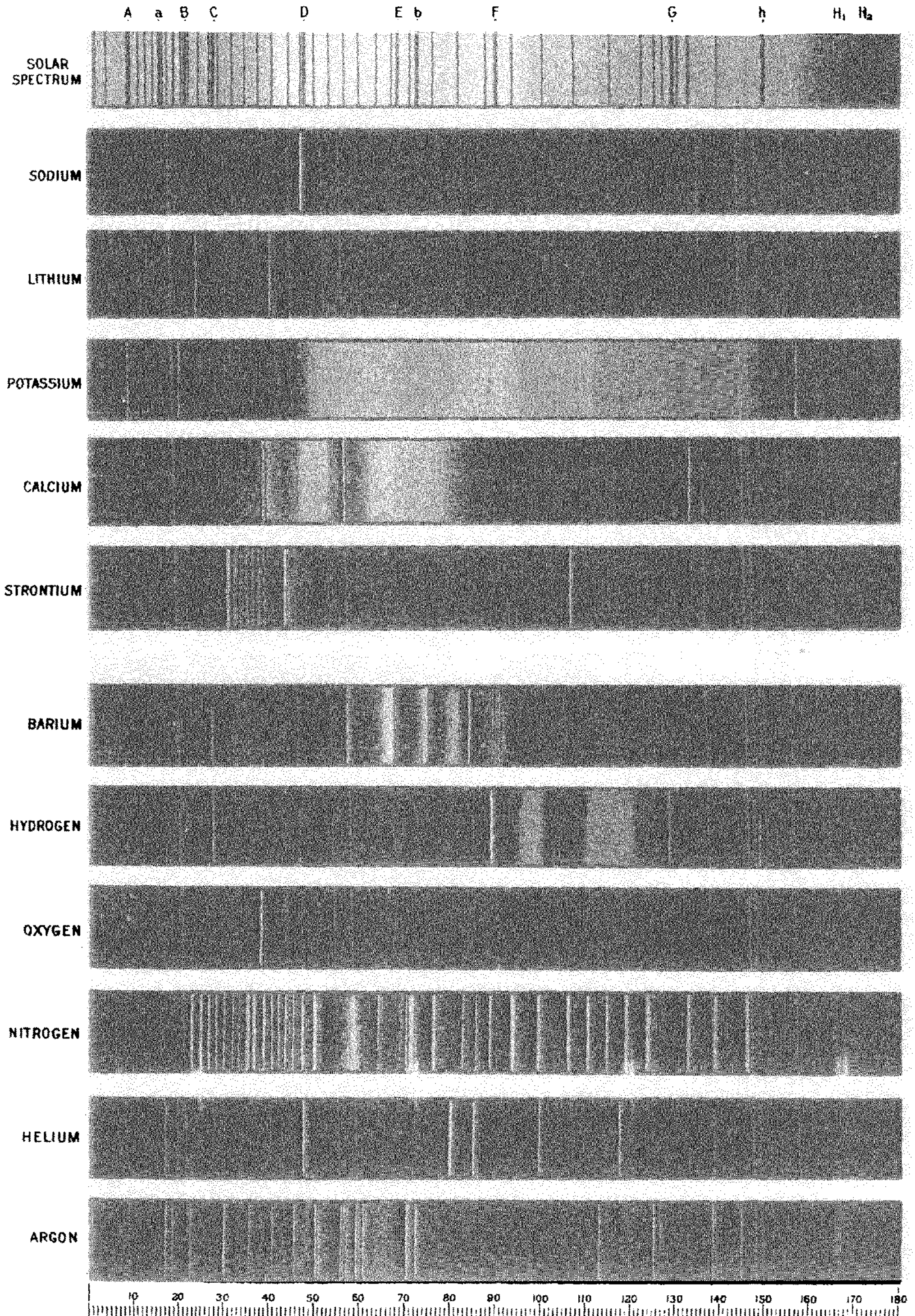

PREFACE.

THIS Treatise on Natural Philosophy differs from others in the large number of practical experiments and exercises which it contains. The authors believe that students of science should be, as far as possible, investigators, and, to encourage the spirit of research, they have given suggestions tending to lead them on in this way. The experiments can nearly all be performed with very simple and inexpensive materials, such as any school or home can furnish. More elaborate instruments are described for the benefit of classes which have access to them. The book can also be used by classes which have not time to perform the experiments. Yet it is strongly recommended that as many as possible be tried.

Two sizes of type are used through the book. The matter printed in large type will form a complete elementary course, and the whole book a more exhaustive one. Those who take the former are advised to include as many as convenient of the experiments, exercises, and questions. The large number given will allow the teacher to make selections suited to the ability of the class.

The use of technical terms, except where they seemed necessary to the better comprehension of the subject, has been avoided. It has been recognized that the majority of students of natural philosophy have no use for these terms. What they want is a practical knowledge of the subject and the cultivation of scientific habits of mind.

The methods of the leading scientific men of the present time have been incorporated, and their instruments described and figured. In any treatise on the subject which embraces an account of these methods, the doctrine of the conservation of energy must have a prominent place. The great advances in practical science within the last few years, especially in sound, electricity, and meteorology, have also been utilized so far as they seem to bear on the principles.

NOTE TO REVISED EDITION.

In the present revision, made by C. Canby Balderston, of Westtown School, Pennsylvania, the chapter on Electricity has been entirely rewritten, with a view of making it represent as nearly as possible the present state of the science. The chapters on Matter, Motion and Force, and Light have been largely rewritten and rearranged, experience in the use of the book having suggested some changes. The subject of surface-tension has been introduced into the chapter on Liquids, and the naphtha-engine is described in the chapter on Heat.

As a new feature, a summary of each chapter has been added, which cannot fail to assist the student in gaining a clear conception of the subject.

CONTENTS.

NATURAL PHILOSOPHY.

CHAPTER I.

MATTER.

1. **What is Matter?**—All the bodies and substances which occupy space, the stars and the planets, rocks, water, and air, and everything we can see or feel, is composed of *matter.*

We can crumble a rock or divide a quantity of water into smaller portions. These can again be subdivided, and all the fragments will resemble the original in their properties. There is a practical limit to this subdivision, arising from the imperfection of our senses or our tools, but we may *suppose* it carried on till the very smallest possible fragments remain which possess the properties of the substance.

2. **Molecules.**—To these fragments we give the name molecules. They are definite quantities of matter, which have size and weight.

Hence *a molecule is the smallest portion of any substance in which its properties reside.*[1] All matter is made up of molecules. We know that molecules must be extremely small. Sixteen ounces of gold, which in the form of a cube would not measure an inch and a quarter on a side, can be spread out so that it would gild silver wire sufficient to reach round the earth. Its thickness must then be at least

[1] The properties of matter are those qualities which are peculiar to it,—which belong to it and to nothing else.

one molecule, and is doubtless many. In odors, which produce sensation by invisible particles, the molecules scatter about through the atmosphere for years without apparently diminishing the size of the substance from which they are separated. Microscopists have found animals so minute that four million of them would not be so large as a single grain of sand, yet each has its organs and its circulating fluids.

3. **Size of Molecules.**—No molecule of any substance is large enough to be visible, even in the most powerful microscope. It is only by the most careful experiments that any notion of the size of molecules can be formed, and the results of such experiments indicate that they are minute beyond our comprehension. Sir William Thomson estimates that if a drop of water as large as a pea were magnified to the size of the earth the molecules would then appear scarcely larger than the original drop.

The spaces between the molecules of matter are believed to be much larger than the molecules themselves.

4. **Atoms.**—It is not possible to divide a molecule by pounding or grinding, but by means of heat or some other chemical agent most molecules may be separated into two or more portions. Each of these is called an atom; and this cannot be further divided. The word atom means indivisible.

5. **Elements and Compounds.**—The atoms of some substances are all alike,—that is, the substance is composed of one kind of matter only, which cannot be separated into other kinds. Such a substance is called an *element*. About seventy elements have been found on the earth. Examples: iron, gold, silver, carbon, hydrogen, oxygen. Most of the substances which we see are *compounds* of two or more of the elements. Water is composed of hydrogen and oxygen; wood, cotton, starch, sugar, are composed of hydrogen, oxygen, and carbon. Salt is composed of chlorine and sodium.

6. The elements in a compound are generally very differ-

ent from each other and from the compound. The science of chemistry treats more fully of the elements and their union into compounds. Here are two chemical experiments.

Experiment 1.—Dissolve a little baking- or washing-soda in water and pour in some vinegar. Bubbles of gas come out.

A molecule of soda is composed of a number of atoms of different substances. The acid in the vinegar causes a division of the molecule, forming new substances. One of these substances (carbonic acid) is a gas, which passes off into the air. The others remain in the vessel.

If a small portion of sugar be burned on a stove, a black substance will remain. A molecule of sugar is composed of forty-five atoms of three different kinds,—carbon which we can see as charcoal, and hydrogen and oxygen, which are colorless invisible gases. Heat separated the atoms of the molecules; the gases passed off into the air, and the solid carbon remained.

7. **Matter Indestructible.**—If the escaping gases and the carbon of the last experiment could be weighed, the sum of the weights would be found to be just equal to the weight of the original sugar. Hence we arrive at an important property of matter,—*it is indestructible.*

There are many cases of the apparent destruction of matter in combustion and chemical action, but all that is done is to change its form. The molecules are divided, and the atoms form new combinations, some or all of which are invisible. In all the various changes continually going on, in our furnaces and laboratories, and in nature, not a new atom is ever created. According to the best of our knowledge, the amount of matter in the universe has remained unchanged since the original creation.

8. **Matter Porous.**—The molecules of matter do not fit closely together. Hence open spaces, or pores, are left between them. We then arrive at a property of matter which is believed to be universal,—*it is porous.*

Experiment 2.—Fill a tumbler with cotton-wool, pressing it down so firmly that the vessel will hold no more. Now remove the cotton and fill the vessel with alcohol. With care, the cotton may all be replaced without spilling the alcohol. The cotton has gone into the pores of the alcohol, and the alcohol into the pores of the cotton. It is impossible to conceive that the molecules of both substances occupy the same space.

9. **Matter can be Expanded and Compressed.**—As a result of the porosity of matter, it is possible to *expand* or to *compress* it. The molecules are not changed in form or size, but they are further separated in expansion, and crowded together in contraction, so that the substance becomes more porous in one case and less so in the other. Heat in general separates the molecules from one another. A ball that will just go through a ring when cold will not do so when heated. The mercury in a thermometer-tube rises in hot weather because the heat separates the molecules and there is no chance for expansion in any other direction. The ends of the rails of a railroad-track which touch each other in summer are separated in winter. A nail can be driven into wood because it causes a compression of the molecules around to make a place for it.

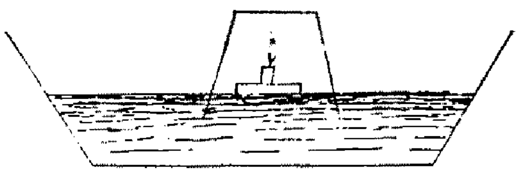
FIG. 1.—EXPANSION BY HEAT.

Experiment 3.—On a cork floating on water place a shaving. Set it on fire, and put over it an inverted tumbler. The heat of the combustion will expand the air in the tumbler and force it out under the edge; what is left will quickly cool and contract, so that almost immediately the water will rise into the tumbler.

10. **Malleability and Ductility.**—Malleable substances are such as can be hammered or rolled into sheets. Ductile substances are such as can be drawn into wire. In these cases the molecules slide past one another and arrange themselves differently. This motion of the molecules is not possible in all solid bodies, and some possess it in a much higher degree than others. Gold may be hammered out into sheets less than $\frac{1}{200,000}$ of an inch in thickness. Copper, silver, and tin can also be beaten out into very thin foil.

One of the substances which may most readily be drawn out into wires is glass. Heated red-hot in an alcohol flame or hot gas flame, a small glass rod or tube may be drawn out into a very fine thread.

Metal wire is made by drawing the soft metal through holes, each one smaller than the preceding. Platinum wire can be reduced so that it will be finer than the finest hair.

11. **Elasticity.**—Almost all bodies and all substances are more or less *elastic;* that is, when forced into a different size or shape,—some by compression, some by stretching, some by bending or twisting,—the molecules tend to resume their positions with reference to one another, and restore the substance to its shape or size.

Experiment 4.—Let a rubber ball fall on a dusty, varnished table-top. Notice the size of the mark it makes. Lay it quietly on another spot of the table, and compare the marks.

12. When a ball is allowed to fall on a hard floor, there is a compression of the molecules of the ball near the point of contact with the floor. The elasticity of the ball causes an immediate restoration to the original form of the ball, and this produces the rebound. When gases are compressed, they recover their former state immediately when the pressure is withdrawn. They are said to be perfectly elastic. Although liquids can be compressed but slightly, they are also perfectly elastic.

Gases exhibit elasticity only on being compressed. Liquids and most elastic solids exhibit it when released from any kind of strain, though some highly elastic bodies can be *stretched* very little; *e.g.*, cold glass, steel.

Query.—What substance is uppermost in your minds as highly elastic when compressed, stretched, bent, or twisted?

13. **Limit of Elasticity.**—As stated above, there appears to be no limit to the elasticity of gases and liquids. Solids reach the limit of their elasticity when the molecules are forced so far from their position that they cannot regain it, and the body remains bent or compressed; or when they actually let go and the body breaks.

Mention some substances to illustrate each limit.

14. **Tenacity.**—When a solid is composed of molecules which adhere so closely that they strongly resist a force tending to pull them apart, it is said to be *tenacious*. The amount of tenacity depends on the structure of the substance. Wrought iron, being fibrous, has much more tenacity than cast iron, which is granular. Steel is very tenacious. A bundle of wires will support much more weight than the same material in solid form. Hence the cables of suspension-bridges, which have to hold up immense weights, are usually made up of bundles of fine steel wire.

If a piece of stick be placed on two supports some distance apart, and broken by a weight applied in the middle, the lower fibres will be found to be separated.

15. **Bridges.**—When a weight rests on a bridge, it has to stand the same kind of strain as the stick. The tendency is to pull it apart at the bottom. Hence an iron bridge has its lower "chord" made of tenacious wrought iron rather than of cast iron. The upper chord is compressed, and is frequently made of cast iron, which will withstand more compressing force than wrought iron will.

16. **Hardness.**—Hardness is a property of solid bodies, which enables them to resist attempts to cut or scratch them.

Experiment 5.—Scratch a piece of glass with the edge of a quartz crystal or piece of flint. Attempt to do the same with a penknife-blade. Quartz is harder than glass, and glass is harder than the knife.

17. **Density.**—The density of a substance is the amount of matter in a given bulk. It is determined by the *weight* of the substance. A cubic inch of lead is heavier than a cubic inch of wood because it is more dense. Density is entirely distinct from hardness.

18. **Volume.**—The volume of a body is the amount of space it occupies, or its size.

19. **Mass.**—The mass of a body is the total quantity of matter which it contains. If a gas be heated it expands,

and the density decreases, but, as no new molecules are formed, the mass remains the same. The mass, therefore, depends on two things, volume and density. The number of molecules in a cubic inch of a body, multiplied by the number of cubic inches, gives the whole number of molecules. In other words, the product of the volume and the density gives the mass, or

$$\text{Mass} = \text{volume} \times \text{density}.$$

20. **Units.**—To designate masses, as indeed to designate length, area, volume, density, time, work, or anything else by number or definite quantity, we must refer in each case to a *unit*. A unit, when it is a measure of size or duration, is simply *one*. Other units are taken as one for *standards of comparison*. For instance, a given volume of hydrogen is lighter than the same volume of any other gas, therefore it is called one, and other gases are compared with it by number to denote their density, oxygen being 16, nitrogen 14, and so on.

21. **Two Systems of Units.**—Most of the countries of Europe have adopted a system of measures and weights which originated in France nearly a hundred years ago. In this system the number of one denomination required to make one of the next higher is always *ten*. As our *numbers* are formed in the same way (units, tens, hundreds, etc.) there is no reduction required when we use this system. It is known as the French or *decimal* system of weights and measures. The use of this system was authorized in this country by act of Congress in 1866, but it was not made compulsory, so in trade we still generally use the old system, which came from England and is still in use there, and which is known as the English system.

22. In this book both systems are used, the English because most boys and girls are more familiar with it, and the decimal, because it is much easier to calculate in, and because it is desirable that it should become familiar to us, so that we may adopt it.

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23. **Unit of Length.**—The English unit of length is the *yard.* We may use the divisions and multiples of the yard —feet, inches, miles, etc.—in certain cases, and they become the units in those cases. The decimal unit of length is the *metre.* Remember that a metre is about 1 yard $3\frac{3}{8}$ inches, a decimetre about 4 inches, and a centimetre about $\frac{2}{5}$ of an inch. (See Appendix.)

24. The standard yard is a metal rod preserved in the Royal Exchequer, London. Other yard-sticks are as nearly the length of that one as they can be made. The standard metre is a metal rod preserved by the government of France. It is intended to be one forty-millionth of the meridian circumference of the earth.

25. **Units of Surface and Volume.**—The units of surface are simply the *squares* of the units of length, and the units of volume the *cubes* of the units of length. For instance, square foot, square yard, square metre; cubic foot, cubic yard, cubic metre. The square decametre and cubic metre take new names, as units, the square decametre being an *are* and the cubic metre being a *stere.* (Pronounced air and stair.) The cubic decimetre is the *litre* (leeter), and is the unit of volume for liquids, grain, etc.

26. **Unit of Mass.**—The English unit of mass is the avoirdupois *pound.* The French unit is the *gram,* which is the mass of a cubic centimetre of water at its greatest density. The pound is simply a standard weight of metal preserved in the Royal Exchequer.

27. **The C. G. S. System.**—The decimal system of units is generally designated in more advanced books as the "C. G. S." system, from the units of length, mass, and time,—centimetre, gram, second.

28. **Unit of Density.**—The unit of density for solids and liquids is the density of water at 39.2° F.

29. **Affinity, Cohesion, Attraction.**—The force which holds together the atoms in a molecule is called *affinity.*

The force which holds together the molecules in a body is called *cohesion.*

The force which holds together the different bodies of the universe is called *attraction.*

Hence affinity makes *substances;* cohesion makes *bodies;* attraction makes *systems.*

Attraction is also used to express the force which draws one body to another, as in the case of magnets, etc.

30. **States of Matter.**—There are three states or conditions of matter,—solid, liquid, and gaseous. A possible fourth state of matter is referred to in Arts. 45 and 570.

31. **Solids.**—In *solids* the molecules preserve their positions with considerable firmness, resisting attempts to displace them. Hence these retain their form and size. The force of cohesion is strong.

32. **Liquids.**—In *liquids* there is perfect freedom of the molecules among themselves, so that the bodies adapt their form to the surrounding vessel. They retain their size, but change their form with the slightest force exerted upon them. The force of cohesion is weak.

33. **Gases.**—In *gases* there is no cohesion, the molecules have a *repellent* action upon one another, so that an unrestrained gas will expand indefinitely.

34. **Motion of Molecules.**—The molecules of all bodies are believed to be in rapid motion. In solids this is restrained by cohesion, so that a molecule has only a short vibratory motion. In liquids the molecules slide over one another without resistance, restrained only when they reach the sides of the enclosing vessel. This contact produces the pressure against the sides. In gases the molecules are strongly repelled from one another, and dash about with great velocity. Hence there are constant collisions among them and with other bodies. Our bodies are subject to this incessant battering by the little molecules of the atmosphere, but, the force being the same on both sides of the tissues, we do not notice it.

35. **Adhesion.**—Adhesion is the force with which different surfaces stick together. It is adhesion which causes mortar to stick to bricks, paste to stick to paper, glue to wood, or two glued surfaces to stick to each other.

36. **Gravity and Weight.**—Unsupported bodies fall towards the earth. This is on account of the earth's attraction for them. Bodies that are supported press downward on account of the same attraction. The greater the attraction the harder the pressure. The *weight* of a body is simply the measure of the earth's attraction for it.

37. **Weight Proportional to Mass.**—The earth attracts or pulls every particle of a body. Suppose the pull of the earth on each particle to be exerted through a string attached to the particle, and suppose all the strings to be pulled together, the sum of all the pulls would represent the attraction on the whole body. Hence the more molecules the greater the attraction. But the mass is determined by the number of molecules. Hence we have the law,—

Under the same conditions the weights of bodies are proportioned to their masses.

38. **Unit of Weight.**—The unit of weight is the same as the unit of mass, the pound or the gram.

39. **How Gravitation Acts.**—The earth's attraction for bodies on or near it has been called gravity. In a wider sense, applied to the universe, it is called gravitation. This force is in many ways different from other forces. It does not require any time to act, nor does it require any medium to act through. It traverses the great space between the sun and the earth, to the best of our knowledge, instantaneously. Nor does the interposition of another body affect it in any way. We can cut off sound, or heat, or light, by the interposition of a wall, but attraction acts through it without diminution. Nor does the kind of matter make any difference. Every molecule is attracted alike, the number of molecules determining the total attraction.

40. **Law of Gravitation.**—The main facts of gravitation were discovered by Sir Isaac Newton, who announced the following law: *Every particle of matter in the universe at-*

tracts every other particle, the attraction of any two for each other being directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

Questions.—How much is the attraction between two bodies increased by doubling the mass of one of them? By doubling the mass of both? How is attraction affected by doubling the distance between two bodies? By doubling both masses and doubling the distance between them?

41. **Gravity Above and Below the Earth's Surface.**—Gravity is greatest at the surface of the earth. When we go down into the earth gravity decreases, because some of the matter of the earth is attracting us upward. Were we to get half-way to the centre we should have only half the weight that we have at the surface. At the centre we should have no weight, being equally attracted in all directions.

As we go above the earth gravity decreases as the square of the distance from the centre of the earth increases.

42. **Mass Constant.**—The position of the body does not affect the *mass.* It might be removed far from the earth and the mass would be the same. The number of molecules—*i.e.*, the mass—would be constant if carried to the sun; but as there is so much more mass in the sun than in the earth, the attraction, and consequently the weight of the body, would be greatly increased.

43. **Mobility and Inertia.**—Bodies will not move unless some force is exerted on them from without, and they yield to the slightest force impressed which is not counterbalanced by some other force. This brings us to two other properties of matter,—*mobility*, which induces it to yield freely to impressed forces, and *inertia*, which prevents it from *moving* itself, from *stopping* itself, or from *changing its direction* of motion.

Examples of inertia are numerous. It requires more force to start a car than to keep it in motion. When suddenly stopped by another force, the contents are thrown forward by their inertia. A ball projected upward stops, not because it has power to stop itself, but because another force, gravity, is constantly pulling against its motion. A marble thrown swiftly through a pane of glass will make a small round hole, because the inertia of the other parts of the glass prevents them from yielding to the sudden impression.

Experiment 6.—Place a card on the end of a finger, and a cent on the card. By a quick stroke with the forefinger of the other hand the card may be shot out, leaving the cent resting on the finger.

44. **Ether.**—We have spoken of the three forms of matter, solid, liquid, and gaseous; we have also said that the molecules of matter do not fill up the whole space, but that pores, which are large compared with the size of the molecules themselves, exist in all substances. This intermolecular space is supposed to be filled with something called *ether*, which is as far separated from gases by its properties as gases are from liquids. It also fills the pores of the air, and the spaces between the planets and between the stars, outside the bounds of the atmospheres which surround them. It is highly elastic, without weight or color, or any other properties which can be perceived by the senses. It is supposed to be the agent which by its vibratory motion conveys the rays of light from the sun to the earth, and which carries them between the molecules through transparent substances.

45. **Radiant Matter.**—Dr. William Crookes[1] has found that by exhausting the air in a tube so as to leave not more than one-millionth the ordinary amount, the remaining substance has properties so peculiar that he feels justified in giving it a new name. He calls it *radiant matter*, and considers it to be a fourth form of matter. Solid, liquid, gaseous, and radiant would then be the four aggregate

[1] An English scientist, now living (1892).

states, each having properties which widely separate it from the others. By passing electric sparks through radiant matter some of its properties have been determined.[1]

Exercises.—1. Is matter destroyed when water is dried up? when gunpowder explodes? when house gas burns? Where does it go to?

2. To what property of matter do blotting-pads owe their utility? rubber bands? watch-springs? pop-guns? putty? hammers? piano-strings? water-filters?

3. Why does not the addition of a little sugar to a full cup of coffee cause it to overflow?

4. When we fix the head of a hammer on the handle by striking the end of the handle on a block, what property do we use?

5. Why does a foot-ball, nearly empty, become full when we exhaust the air from around it? why does it soon collapse?

6. One sixteen-thousandth of a cubic inch of indigo dissolved in sulphuric acid can color two gallons of water. What property of matter is here shown?

7. How would you test the relative hardness of two minerals?

8. When water is converted into steam, are the molecules enlarged or separated? is its mass increased or diminished? its density? its weight? its volume?

9. Name a substance which is often found in all three forms.

10. If you knew the volume and mass of a solid, how would you obtain its density? if you knew its mass and density, how would you obtain its volume?

11. Give an instance of a hard body which has little cohesion.

12. Why does not a large stone fall to the earth more rapidly than a small one?

13. If a body were removed to a distance of 8000 miles from the surface of the earth, how much less would it weigh than at the surface? *Ans.* $\frac{1}{9}$ as much.

14. What would a 100-pound weight weigh if moved to the distance of the moon (60 radii of the earth)? *Ans.* $\frac{1}{36}$ pound.

15. Suppose a sphere were one-half the diameter of the earth and of the same density, what would a body which weighed 100 pounds on the earth weigh at its surface? *Ans.* 50 pounds.

Note.—Its mass would be one-eighth that of the earth, and distance of the body from its centre one-half.

SUMMARY OF CHAPTER I.

46. **Matter** is made up of a countless number of minute molecules. It is perfectly inert, but each particle has the property of attracting every other particle.

It has extension in three dimensions. It has three (possibly four) states of aggregation.

[1] These will be further explained, page 33".

CHAPTER II.

MOTION AND FORCE.

47. **Rest and Motion.**—In natural philosophy we generally speak of *motion* with reference to fixed points on the earth, and these are said to be at *rest*, though of course they are moving with the earth through space. We may consider motion with reference to any other point. A man moves in a boat when he shifts on his seat, or in a moving train when he walks from one part of the train to another.

48. **Kinds of Motion.**—When a body in motion passes over equal spaces in equal times, its motion is *uniform.* When it passes over unequal spaces in equal times, its motion is *varied.* When the spaces in successive times become greater, its motion is *accelerated*, and when less, *retarded.* This acceleration or retardation may also be uniform or varied.

49. **Velocity.**—The *velocity* of a motion is the space traversed in a unit of time. It may be in miles per hour, feet per second, etc.

Feet moved in Successive Seconds.				Kinds of Motion.
30	30	30	30	Uniform.
10	15	20	25	Uniformly accelerated.
20	18	16	14	Uniformly retarded.
20	14	16	4	Varied,—not uniformly.

Questions.—When a train starts from a station, what kind of motion is it? when stopping? when a ball is thrown upward? when it falls? What kind of motion in the hands of a watch? in the current of a river? in the winds?

50. **Force.**—*Force is anything which tends to produce, change, or destroy motion.* One force on a body at rest tends to move it. If it acts on a body in motion, it may change the direction or velocity of the motion, or destroy it. Two or more forces may act on a body at rest so as to balance each other and cause no motion. But each one *tends* to produce motion. In bridges and buildings we have cases of balanced forces. Gravity is a force always acting upon them, and upon everything they sustain. This produces other forces acting along the various timbers and pieces. If the structure is well built, the strains from these forces are exactly balanced, every part is sufficiently strong to do its work, and there is no motion except such as is due to the elasticity of the materials.

51. **Kinds of Force.**—A force may act for an instant and then cease, in which case it is said to be an *impulsive* force; or it may act for some time, when it is a *continuous* force. The striking of a ball by a bat is an example of an impulsive force, and the pulling of a train by a locomotive, of a continuous force.

52. **Impulsive Force produces Uniform Motion.**—*An impulsive force tends to produce uniform motion, and a continuous force accelerated motion.* This would seem to be contradicted by experience. For the motion of a ball is soon destroyed, and the continual pull of the engine may only keep the train moving uniformly. But the force of the bat or of the locomotive does not act alone. Were it not for gravity, the resistance of the air, and friction, which are modifying forces, the ball would move on forever with uniform velocity, and the velocity of the train would be accelerated so long as the engine pulled it ever so slightly.

53. **Newton's Laws of Motion.**—All the circumstances of motion are embraced in three laws, first enunciated by Sir Isaac Newton. These cannot be proved mathematically. They should be looked upon as fundamental prin-

ciples, which depend on the properties of matter, and which may be shown to be true by experiment.

They are as follows:

1. *A body at rest remains at rest unless acted on by a force; and a body in motion would move forever in a straight line with uniform velocity unless acted on by a force.*

2. *Motion, or change of motion, is proportional to the force impressed, and is in the direction in which the force acts.*

3. *For every action there is a reaction equal in amount and opposite in direction.*

54. The first law is the result of the inertia of matter, and the second, of its mobility. The first says matter can do nothing itself, and the second, that the slightest force will have its corresponding effect.

The third law may be made clear by some illustrations. The earth attracts an apple and causes it to fall. The apple attracts the earth just as strongly, and the earth moves to meet it, but the greater mass of the earth makes it move so little that the motion is not noticed. A horse walking on a tow-path pulls a loaded canal-boat. He acts, through the rope, on the boat, and the reaction occurs where his feet press backward against the ground. Put the horse on the forward deck and let him pull ever so hard on the same rope shortened, he could not move the boat, because both action and reaction would be exerted on it, and would balance each other.

55. **Momentum.**—*Momentum is the quantity of motion.* The momentum of the earth *was the same as the momentum of the apple.* For while its *velocity* was less, its *mass* was as many times greater. Hence mass and velocity together make up momentum. A body weighing two pounds has twice the motion of one of one pound which has the same velocity; a body with twice the velocity of another has twice the motion, the mass being the same. In general we have the equation,—

$$\text{Momentum} = \text{mass} \times \text{velocity}.$$

56. The number representing the momentum of a moving body has no name, and such numbers are used only in comparing. A body weighing 200 pounds and moving 10 feet per second (momentum 2000) has twice the momentum—that is, twice the moving power—of a body weighing 50 pounds and moving 20 feet per second (momentum 1000). In the case of action and reaction between two moving bodies, or bodies free to move towards each other, the momentum of one is equal to the momentum of the other.

Questions.—A man standing in the bow of a boat which, with its load, weighs 500 pounds, pulls on a rope which extends to a man standing in the bow of a similar boat weighing 1000 pounds. How do the momenta of the boats compare? How do their velocities compare?

57. **Measure of Forces.**—We may measure forces in two ways. One way is by the pressure necessary to resist them,—*weighing* the forces, as it were. The unit is then the pound or the gram. These vary as gravity varies, being greater nearer the level of the sea. A better way to measure forces is by the velocity they would produce if acting alone. The velocity which a force can impart to a unit of mass by acting on it for a unit of time is its *acceleration.*

58. **Units of Force.**—In the English system the units are the pound and second, and the unit of force is that force which in one second will give a mass of one pound a velocity of one foot a second. The C. G. S. system employs the C. G. S. units (Art. 27), and gives us the unit of force called the *dyne*, which means force. The dyne is the force which, acting for one second, will give a mass of one gram a velocity of one centimetre a second, *i. e.*, which will give one gram an acceleration of one centimetre per second.

59. If a force of a dyne act constantly on a moving mass of one gram, it will increase its velocity regularly, for at the end of each second the inertia keeps it moving at its acquired velocity, and the force *adds* to its velocity continually. The additional velocity in any mass is directly proportional to the force; hence the *acceleration is a measure of the force.*

60. **Magnitude of the Units of Force.**—The units of force must not in any way be confounded with the units of weight which

are named in defining them. The *weight* of a pound or of a gram is a measure of gravity. Now, gravity acting on any ordinary mass of matter near the earth and free to fall, gives it an acceleration of about 32.2 feet, or 980 centimetres per second. (See Art. 88.) If the mass is a gram, and it acquires a velocity of 980 centimetres a second, it must be acted upon by a force of 980 dynes, as one dyne could give it a velocity of only one centimetre per second. The dyne is equivalent, then, to a force or pressure of $\frac{1}{980}$ of a gram at the sea-level. The *megadyne* (million dynes) is used in practice to denote forces of appreciable magnitude.

61. Effect of Two Forces.—**Experiment 7.**—Fasten two pulleys against a vertical board so that they will turn freely. Arrange cords as in the figure, making the knot at *a* so as not to slip. Hang weights *p*, *q*, and *r*, being careful not to get *r* greater than *p* and *q* combined. Measure *ab* and *ac*, proportional to the weights *p* and *q*; that is, if *p* is three pounds, make *ab* three feet, and if *q* is four pounds, make *ac* four feet. Make the string *bdc* seven feet long, and fasten it at *b* and *c*. Put a pin at *d*, four feet from *b* and three feet from *c*. Hang a plumb-line from this pin. It will pass the point *a* and the weight *r*, and the distance *da* will be as many feet as there are pounds in *r*.

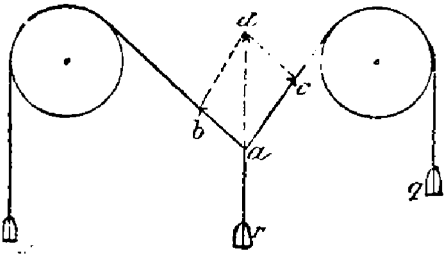

FIG. 2.—PARALLELOGRAM OF FORCES

62. Parallelogram of Forces. — The figure *abdc* is a *parallelogram*, and the line *da* is the *diagonal*. Any two forces acting on a body in the same plane, but not in the same straight line, move the body from one corner to the opposite corner of a parallelogram, just as the forces *p* and *q* suspend the weight *r* in the direction of the diagonal. This is illustrated in rowing a boat *across* a current, and in many other familiar ways. Draw two lines to represent the direction and magnitude of the forces. Through the end of each of these lines draw a line parallel to the other. The figure will be the parallelogram of forces, and the

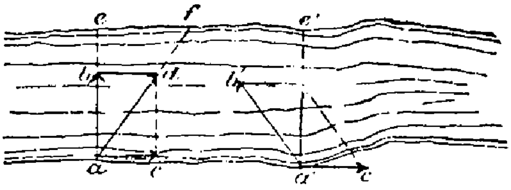

FIG. 3.—CROSSING A CURRENT.

diagonal, which represents the direction and distance taken by the body, is the *resultant* of the two forces. In Figs. 2 and 3, *ad* is the resultant of *ab* and *ac*. So any number of forces may have one resultant.

63. **Centrifugal Force.**—When a body is swung around by a string there are two forces acting on it. One is its inertia, which would tend to make it move in a line, *ab*, touching the curve. The other is *ac*, the pull of the string. The tendency would be to move in the diagonal *ad*. But as this pull is acting continuously, and the direction continually changing, the line is a curve. These are the forces which keep the earth and all the planets in their orbits.

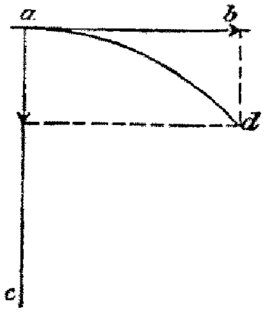

Fig. 4.—Motion in a Curve.

The outward pull on a string, which is the result of the

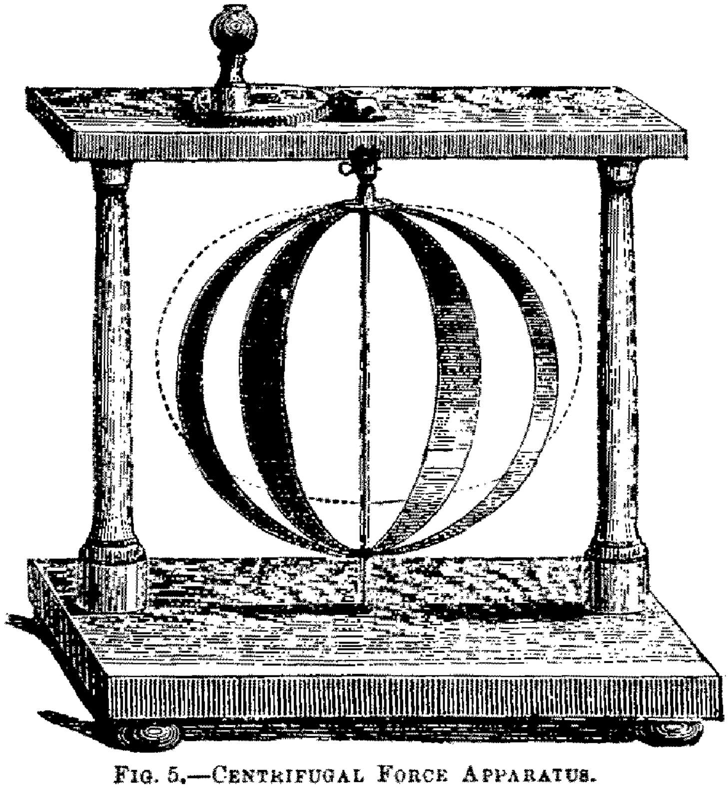
Fig. 5.—Centrifugal Force Apparatus.

inertia of the body tending to cause it to get farther from the centre, is *centrifugal force*. It is always equal to the force drawing towards the centre and opposite in direction.

64. **Effects of Centrifugal Force.**—A striking effect of centrifugal force is shown by the apparatus of Fig. 4. Here the flexible bands are put in rapid rotation, and the centrifugal force makes them assume the form indicated by the dotted line. When the earth was a soft body, the centrifugal force caused by its rotation on its axis probably produced the bulging at the equator which we now notice. The centrifugal force is greater at the equator than elsewhere, because of the greater velocity of the earth there. Hence bodies are lighter there than at the poles. An equestrian leans inward in riding around a curve, to balance the centrifugal force. It is this force which causes mud to fly from moving carriage-wheels, or water from a grindstone, and which sometimes breaks a rapidly-revolving fly-wheel. In sugar-refineries the syrup is separated from the crystals by being thrown outward, the sugar being retained by a wire gauze. Clothes are dried by a similar arrangement. In a bicycle in motion the centrifugal force causes the particles to continue to move in the same plane. Hence the faster it is going the more difficult it is to overturn.

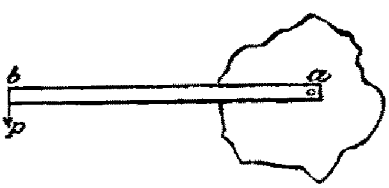

FIG. 6.—MOMENT OF A FORCE.

65. **Moment of a Force.**—The moment of a force is its ability to produce rotation. If *ab*, Fig. 6, be a lever rotating about the point *a*, and a force *p* be applied at *b*, in the direction of the arrow-head, it will, if sufficient, turn the body. This ability will depend on the magnitude of the force and the length of its lever-arm, and is equal to their product. Thus, the moment of $p = p \times ab$.

Exercises.—1. A force of ten pounds has a lever-arm of 2 feet: what is its moment?

Ans.—20 foot-pounds.

2. A force of 16 grams has a lever-arm of 200 metres: what is its moment in kilogram-metres?

66. **Work.**—Work consists in moving against resistance. A horse or an engine does work when it pulls a load, a bird when it propels itself through the air, a man when he lifts up a weight.

Let us take the latter case. When a load is lifted, a certain amount of work is done; when it is lifted twice as high, twice as much work is done, or when the weight is twice as great, twice as much work is done; when twice as great a weight is lifted through three times the height, six times the work is done; or,

$$\text{Work done} = \text{weight} \times \text{height}.$$

In general, the work done by any force is the product of the force and the distance through which it moves.

67. **Unit of Work.**—The unit of work in the English system is the *foot-pound.* It is the work done in lifting the unit of weight (pound) through a unit of height (foot). The French system uses the units kilogram and metre, and the unit of work is the *kilogram-metre.*

68. **Power.**—When *work,* simply, is estimated, the *time* required for it is not taken into account. Ten pounds lifted 10 feet are 100 foot-pounds of work, whether hoisted by an engine in a second or carried up a ladder by a boy in a minute. But it is evident that when *power* is estimated time must be taken into account. An engine which will perform 1000 units of work in a second has twice the power of an engine which requires two seconds to perform the same work.

69. **Unit of Power.**—The unit used in estimating the power of engines, boilers, electric motors, water-wheels, etc., is the *horse-power*, which is the ability to do 33,000 foot-pounds of work in one minute. To find the horse-power of an engine, multiply the weight lifted, or pressure of steam in pounds, by the number of feet moved in one minute, and divide by 33,000. An engine having a piston of 165 square inches, a two-foot stroke, and an average available steam-pressure of 25 pounds per square inch,

develops 75 horse-power when making 150 revolutions per minute.

Exercises.—1. Work out the above.

2. How many foot-pounds of work are done in lifting 20 pounds 10 feet?

3. An engine hoists 2 tons of coal up a 600-foot shaft in 1 minute: what horse-power does it exert?

4. What horse-power is required to perform the work of the last example in 2 minutes? in 5 minutes? in $\frac{1}{2}$ minute?

ENERGY.

70. Energy is Ability to do Work. A moving body has energy. A body lifted up has energy. They can do work in moving or in falling. The units of energy are the same as the units of work.

71. The Erg.—The units of work given in Art. 67 are derived from the *weight* of the pound and the kilogram. Weight depends upon gravity, and as this varies with the distance from the earth's centre, any units derived from it must vary accordingly. This variation on the earth's surface is very slight, and the units are likely to remain in use when estimating heavy work. For the purpose of exact science, however, an *invariable* unit of energy is desirable. The *erg* is such a unit. It is the energy expended or the work done by a force of one dyne (Art. 58) acting through one centimetre.

This is equivalent to lifting $\frac{1}{980}$ of a gram, at the sea-level, 1 centimetre high. (Art. 60.)

How many ergs in a kilogram-metre?

72. The dyne and the erg are invariable, because they depend upon the *mass*, not the *weight*, of a cubic centimetre of water.

73. Potential Energy.—A weight held up by the hand has the power by virtue of its position to fall, and hence do work, if its support be withdrawn. A body of water held up by a dam has the power to do work on a water-wheel, if allowed to fall upon it. A wound-up spring has power to perform work in turning the machinery of a clock.

The ability to do work which a body thus has by reason of its position is called *energy of position*, or *potential energy*.

74. **Kinetic Energy.**—A weight descending, water falling on a wheel, a spring uncoiling, a bullet moving through the air, a muscle in use, have energy,—*energy of motion*, or *kinetic* energy, sometimes called *actual energy*.

75. The formula for potential energy is $w \times h$, where w represents the weight of a body, and h the height to which it is raised.

76. **Potential and Kinetic Energy Equal.**—If the body w falls the distance h, it can of course raise a corresponding weight to the same height; that is, its kinetic energy is equal to $w \times$ distance it falls, which is the same as $w \times h$.

77. **Transformation of Energy.**—When a body is thrown upward its energy of motion becomes less and less. When it has reached its greatest height this has been all converted into energy of position. As the body falls the energy of position reappears as energy of motion, and when the body strikes the ground the kinetic energy is equal to the potential energy at the highest point.

78. **Conservation of Energy.**—Energy is indestructible as matter is, though it is frequently changed in form. The heat of a steam-engine is converted into motion, and the motion, carried to a dynamo, is converted into electricity, which may be converted into light or into heat again. So with every form of energy, none of it can be completely destroyed any more than matter can. This great principle is known as the *conservation of energy*, and in its fullest sense it means that the *total energy of the universe is always constant in quantity*, never more, never less, no matter what variations it passes through. When a moving object comes to rest without doing manifest work, its energy is simply transferred to something else, to motion of the air, heat of brakes, bearings, &c.

Experiment 8.—Place about a half-dozen glass "marbles," or better, ivory balls, over a crack between two smooth, level floor-boards. See that they are all in contact. Draw back the ball at one end and roll it against the end of the line. The ball at the other end will move away, and the others will maintain their places. How did the energy pass through the middle balls without moving them?

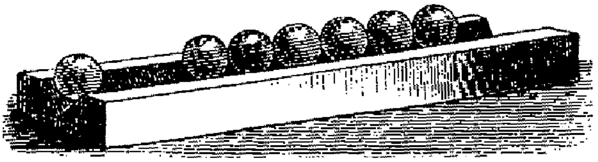

Fig. 7.—Collision Balls.

Experiment 9.—Stretch a string, not too tightly, between two supports several feet apart. Suspend from it, at equal distances from the ends, by equal strings, two equal weights, *a* and *b*, Fig. 8. Draw the weight *b* back, so that it will swing *across* the direction of the horizontal string. Soon *a* will begin to swing, and when it has attained the length of swing originally given to *b*, *b* will be noticed at rest. Immediately, however, *b* starts to swing again, and, on attaining its original height, *a* will be found at rest. So they alternately swing and rest for some time, the friction of the air, etc., finally stopping them. Notice that the original energy of position given to *b* was sufficient to carry either *a* or *b* alone to the full height, or both together to half the height, but never to carry both together to the full height at which *b* started. This is a very entertaining experiment.

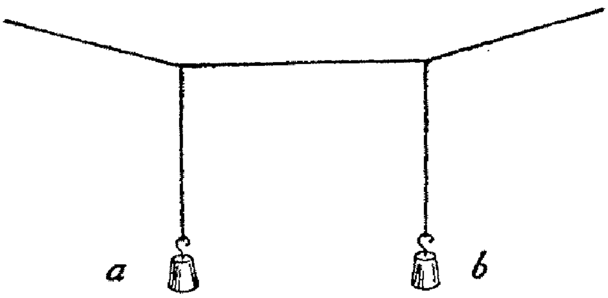

Fig. 8.—Conservation of Energy.

CENTRE OF GRAVITY.

79. **Definition.**—The centre of gravity of a body is the point on which it will balance. If a body has a regular shape and a uniform structure, the centre of gravity is in the middle.

80. **To Find the Centre of Gravity.**—The centre of gravity of an irregular board, or other thin body, may be found as follows: Suspend the body freely on a round awl thrust through it at any point, and suspend a plumb-line from the awl at the same time. The centre of gravity will hang directly *below* the point of support; that is, somewhere in the plumb-line. When the body and the plumb-line come to rest, clasp the line to the body, and mark its path with a pencil and ruler. Now thrust the awl through any

other point of the body and suspend the plumb-line as before. The centre of gravity will be in the plumb-line again, and as it is in both plumb-lines, it is at their intersection. Bore a hole here, and the body will revolve freely on the awl. In the case of most irregular solids the centre of gravity is calculated from the shape of the body and weight of the parts.

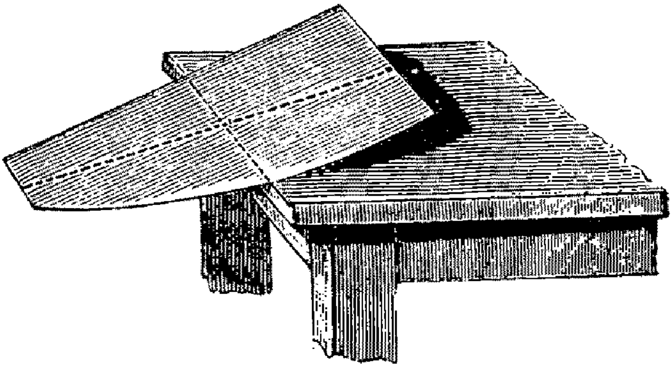

FIG. 9.—CENTRE OF GRAVITY.

Experiment 10.—Lay a thin board on the edge of a table, and when it is just ready to fall, mark the table-edge with a pencil. Change the position of the board, and again mark the table-edge. Bore a hole at the intersection of these lines and insert an awl to rotate the board on.

81. **Line of Direction.**—The line of direction is a vertical or perpendicular line through the centre of gravity of a body.

82. **Stability.**—A body so supported that the centre of gravity is *raised* by any attempt to overturn the body is *stable.* If the centre of gravity begins immediately to *fall* the body is unstable. Example of a stable body, a brick lying flat; of an unstable body, an egg balanced on end.

83. **Base and Stability.**—If the line of direction fall within the base of support of a body, the body will stand, otherwise it will overturn. The larger a body's base the more stable it is, for the line of direction will have more room for play when the body is tilted. The higher a body the less stable it is, for the centre of gravity swings faster

when the body is tilted, and soon brings the line of direction outside the base.

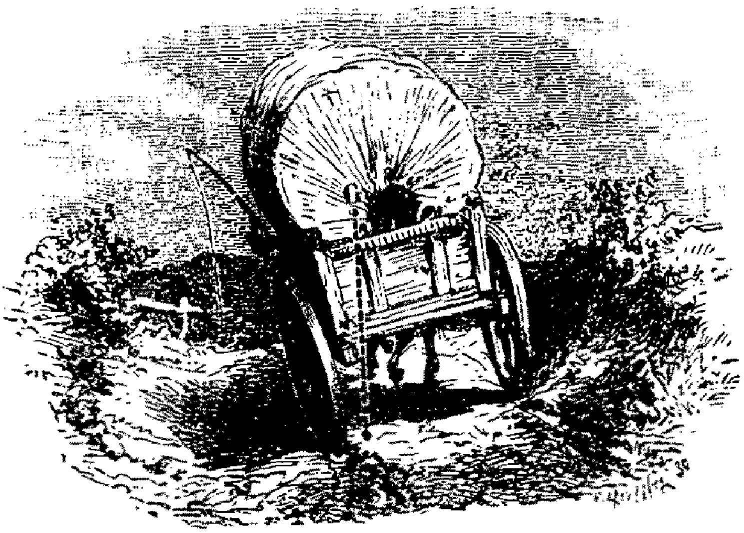

FIG. 10.—LINE FROM CENTRE OF GRAVITY MUST FALL INSIDE THE WHEELS.

Queries.—What would be the effect of building the tower of Pisa[1] considerably higher?

What is the general shape of very high chimneys, and why?

A farmer hauls loads of hay and loads of stone over the same slanting road: which is more likely to upset?

84. When a man stands erect, the line from his centre of gravity falls between his feet. In beginning to walk, he throws his body forward, so as to bring his centre of gravity in front of his feet. He would now fall did he not catch himself by throwing one foot forward. The operation is then repeated with the other foot. He also throws his body from side to side, so as to keep the centre of gravity over the foot which is on the ground. In carrying a weight on his back he leans forward, and in carrying it in one hand he leans sidewise for the same reason.

85. **The Bicycle.**—The bicycle has no base, and therefore it is entirely unstable; that is, it will not *stand*. When running, the rider moves his centre of gravity slightly from side to side, and also by slightly turning his wheel in

[1] Where is this, and how constructed?

the direction in which he tends to fall, he makes use of centrifugal force to straighten up his machine.

86. **Suspended Bodies.**—When a body is freely suspended the centre of gravity is always under the support (Art. 80), and of course it is always stable.

Experiment 11.—Construct the apparatus shown in Fig. 11. *ab* and *bc* are two pine sticks notched together, or hinged together, at *b*. *cd* is a string tied to the handle of the pail and fitting into a notch at *d*. Have the pail part full of corn.

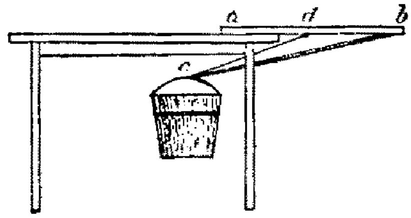

Fig. 11.

Experiment 12.—Bend a piece of rather stout wire into this shape, and fasten an ounce or two of lead to each end. Then place the end of the tongue *T* on the corner of a table, so that the weights will not touch the

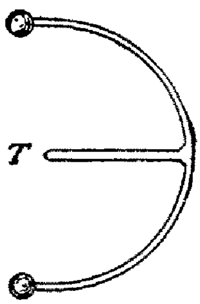

Fig. 12.—Centre of Gravity.

table. On trial, bend the tongue a little, if necessary, so that the frame and weights will rest level.

Experiment 13.—Thrust a needle with a sharp point up through a cork, and put the cork in a bottle. Into another cork thrust a pin and two knives or forks. The side of the pin may now be supported on the point of the needle, and the forks set to revolving, best by *blowing* on them.

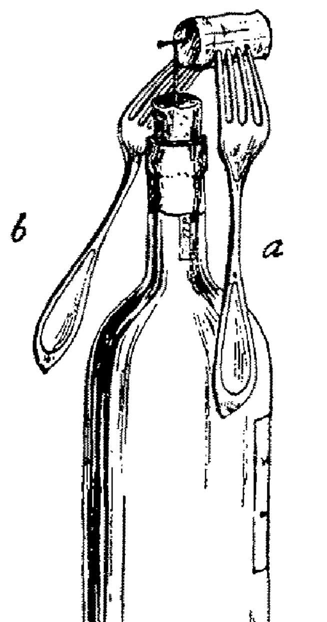

Fig. 13.

87. **Bodies Revolve about the Centre of Gravity.**—**Experiment 14.**—Make a ball of wood or cork and firmly imbed a heavy piece of lead in one side of the ball. Throw the ball up into the air with a twirling motion. Try rolling it across the floor.

Any revolving body, or system of revolving bodies, revolves about the common centre of gravity. We say the earth revolves about the sun. They both revolve about their common centre of gravity, or the point at which they would balance if placed on opposite ends of a lever. (Art. 103.)

Exercises.—1. When will a body slide, and when roll, down an inclined plane?

2. In rising from a chair, why do we lean the body forward?

3. Why is it easier to walk on a fence with a long stick in the hand?

4. When is a pendulum in stable equilibrium?

5. Is a cone balanced on its apex stable? on its base? on its side?

6. Why cannot a person pick up an object from the floor in front of him when standing with his heels against a vertical wall?

7. Should the centre of gravity of a ship be high or low? of a wagon?

8. Why is it easier to suspend an iron ring on a nail on the inside than to balance it on the outside?

9. What would a 200-pound man weigh if moved to within 1000 miles of the centre of the earth? *Ans.* 50 pounds.

FALLING BODIES.

88. **Acceleration of a Falling Body.**—We have learned (Art. 57) that acceleration is a measure of a force. This applies especially to *gravity*, because it acts on a body uniformly—with uniform pressure or stress—no matter in what direction or at what rate it may be moving. Other forces frequently lose in intensity, as the velocity of motion increases, in the direction in which the force acts. This effect of gravity is shown in a body falling towards the earth,—the *centre* of the earth. It is found, by experiment, that a body falling freely towards the earth increases its velocity at the rate of 32.2 feet (9.80 metres) per second. In algebraic equations for solving problems relating to falling bodies this quantity (32.2 feet, or 9.80 metres) is represented by the letter g.

89. It may be asked, Why does acceleration measure the force of gravity more accurately than *weight*, which is the constant pull of gravity on a body at rest? Weight depends upon mass (Art. 37), and as the masses of different bodies are not the same, it is plain that we can derive from this nothing uniform to represent the pull of gravity.

All bodies, small and large, *fall towards the earth with the same velocity*, so that this gives us a uniform standard of attraction.

Some of the old philosophers thought and taught that the rate of a falling body was dependent upon its mass. They did not try the experiment, as any boy or girl may, by letting a dime and a half-dollar drop to the floor at the same instant. The half-dollar is pulled five times as hard as the dime is, and they would have reasoned that therefore it should move five times as fast. They did not consider that the fivefold force has *five times the mass to move*. Suppose we were to let six dimes drop. We should expect them to strike the floor at the same instant. What difference can it make if five of them are joined together in a half-dollar?

90. **Time and Distance of Falling Bodies.**—Having learned by experiment the acceleration caused by gravity, we can easily apply it to finding the distance a body falls in a given time, the time required to fall a given distance, etc. Let us give an illustration:

First, it will be readily granted that as gravity increases the rate of a falling body by 32.2 feet per second, the velocity (represented by v) is always equal to 32.2 feet multiplied by the number of seconds the body has been falling (represented by t, for *time*), or $v = gt$.

Second, it is easily seen that if a body starts from rest and *increases its velocity uniformly* for any length of time, its velocity in the middle of the time will be *one-half* the velocity at the end of the time, or its *average* velocity is one-half its final velocity, or $\frac{1}{2}\,gt$. This average velocity multiplied by the number of seconds must give the whole space traversed by the body (represented by s), or $s = \frac{1}{2}\,gt \times t = \frac{1}{2}\,gt^2$. This equation gives us a direct method of finding times and distances mentioned above.

Note.—These problems may be solved arithmetically. Of course the space divided by $\frac{1}{2}\,g$ (16.1 feet, or 4.90 m.) will give the square of the time in seconds.

Exercises.—1. How far will a body fall in 5 seconds?
2. How far will a body fall in $6\frac{1}{2}$ seconds? *Ans.* 680.225 feet.
3. How many metres will a body fall in 2 seconds?

4. The upper suspension bridge at Niagara Falls is nearly 197.225 feet high: how long would it take a stone to drop from the bridge to the water?

Solution.—As $s = \frac{1}{2}g \times t^2$, $t^2 = s \div \frac{1}{2}g$. $197.225 \div 16.1 = 12.25$, this being t^2, $t = \sqrt{12.25} = 3.5$, number of seconds.

5. The Eiffel tower is 300 metres high: how long would it require for a cold chisel to drop from the top to the bottom. *Ans.* 7.82 seconds. With what velocity would it strike?

6. How high is a balloon when a bag of ballast thrown out requires 9 seconds to reach the earth? What velocity does the bag acquire?

91. **Projection Upward.**—When a body is projected upward, the attraction of the earth takes away from its energy of motion, and when it falls it gives it back again. It has the same velocity in coming down that it had in going up at the same height. The circumstances of the motion are just reversed.

Exercises.—1. A boy kicks a foot-ball straight up. It comes back again in 4 seconds. How high did it go? what velocity did he give it?

Suggestion.—The ball *rose* just half the time.

2. A bullet is shot upward with a velocity of 257.6 feet per second: how high will it go?

92. **Resistance of the Air.**—The figures and results given above make no allowance for the resistance of the air. On account of this resistance the velocity of bodies falling in the air is always less than the calculated velocity, heights are less than calculated heights, and times are longer than calculated times. The resistance of the air is greater for light bodies than for heavy ones, and it increases as velocity increases, but much more rapidly.

Experiment 15.—Carefully cut a piece of paper into a circular shape slightly smaller than a silver dollar. Hold one in each hand and let them drop at the same instant. Now place the paper disk, level, on top of the dollar, and let the dollar go. If carefully done, the dollar pushes the air away, keeps it from the paper, and they fall together.

93. If a body were projected horizontally from the top of a tower, it would reach the level at the same time as if it were dropped. Moreover, it would reach the level at the same time whatever its velocity of projection. For gravity

is the only downward force acting, and it pulls the ball to the earth in just the same time, whether it moves horizontally during the time or not.

Experiment 16.—Slide a coin or roll a marble with force from a level table so that it will shoot horizontally nearly across the room. Just as it leaves the table let another one *drop* to the floor. If accurately started, they will strike the floor at the same instant.

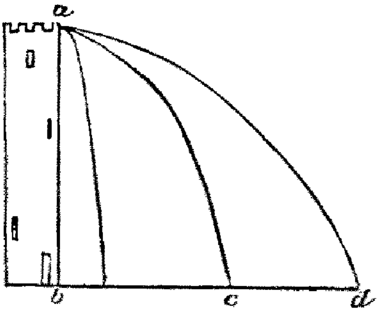

FIG. 14.—PROJECTION HORIZONTALLY.

94. **Curved Path of a Projectile.**—In the above experiments the projected body was acted upon by two forces, the projecting force and gravity. The projecting force was impulsive (Art. 51), but gravity acted continuously. The path was curved. All projectiles move in curved paths because they are acted upon continuously by gravity. It is a law, that *motion in a curve is always produced by the action of a continuous force directed toward a point out of the line of motion.* The grandest examples of this are seen in the motions of the heavenly bodies. The planets revolve around the sun, having received somehow an initial velocity, moving by inertia, and being *continuously* acted on by the sun's gravitation.

THE PENDULUM.

95. **Definition.**—A pendulum is a weight suspended by a cord or rod, so that it may swing back and forth. One swing of a pendulum is called a *vibration.* When drawn aside from a vertical line, the weight is raised and gravity causes it to descend. Its inertia carries it up the other side, and were it not for friction and the resistance of the air it would rise to the height from which it fell, and swing back and forth forever. On account of these resistances it does not rise so high, but makes shorter and shorter vibrations, and is finally brought to rest.

96. **Energy of a Pendulum.**—When drawn aside, it has energy of position equal to its weight multiplied by *ab*;

4

this is converted into energy of motion in the fall, and this is reconverted to energy of position in the ascent, except such portion of it as appears as heat in the point of suspension and in the air. Finally the whole energy is converted into heat, and the pendulum comes to rest.

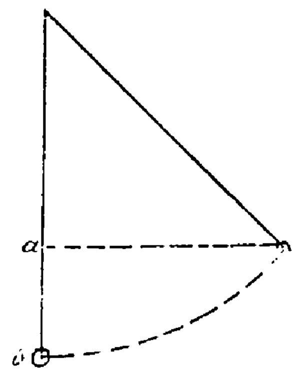

FIG. 15.—PENDULUM.

97. **Laws of the Pendulum.**—1. *The time of a given pendulum is independent of the extent of vibration.*

With the common pendulum, a long swing is performed in very nearly the same time that a short one is. A pendulum can be so arranged that the times will be exact.

2. *The times of different pendulums at the same place are proportional to the square roots of their lengths.*

The converse of this rule is useful in practice; the *lengths* are proportional to the *squares of the times.*

Note.—The length of a pendulum is taken from the point of suspension to the "centre of oscillation," a point near the centre of gravity of the weight.

3. *The times of the same pendulum at different places are inversely proportional to the square root of the intensity of gravity.*

98. This law gives us a means of determining the shape of the earth. A pendulum at the equator is made to beat seconds accurately. Taken to New York it beats more rapidly, showing that it is nearer the centre of the earth. (Art. 64.) Taken to Iceland it beats more rapidly yet, showing that it is still nearer the earth's centre.

Query.—How much nearer is the North Pole to the earth's centre than a point on the equator, both being at sea-level?

99. **The Seconds Pendulum.**—In the Middle United States a pendulum about 39.1 inches, or 993 millimetres,

long vibrates once a second, and is called the *seconds pendulum.*

Experiment 17.—(1st Law.) Suspend a weight of one or two ounces by a thin string to a nail driven into a wall. Slide the loop out on the nail, that the weight may not *rub.* Measure the string about 39.1 inches to the centre of gravity of the weight. Draw the weight aside a foot and let it swing. Take the time of 30 vibrations. Again take the time of 30 vibrations when the pendulum is swinging but a few inches.

Experiment 18.—(2d Law.) Suspend a second weight by a string one-fourth as long as the first. Draw aside and let both go at once. What result?

Exercises.—1. If a seconds pendulum in a given place is 39 inches long, how long is a pendulum that beats half seconds? how long to beat one-third seconds? how long to vibrate in two seconds? how long to vibrate in one minute?

2. Some experiments were recently made with a pendulum at Bunker Hill Monument. If the string was $208\frac{8}{15}$ feet long, what was the time of vibration? (See Art. 99.)

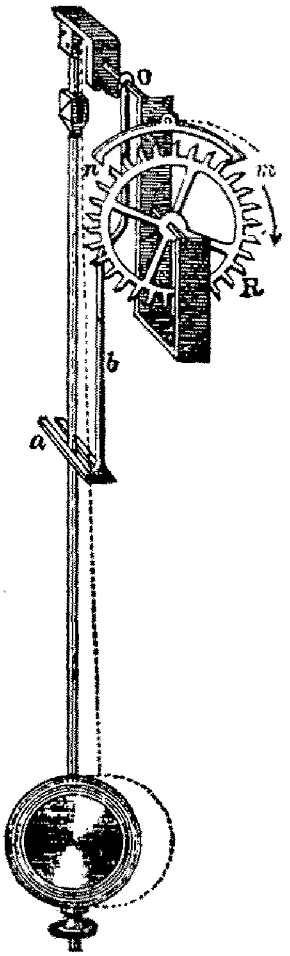

FIG. 16.—PENDULUM FOR CLOCKS.

100. Pendulum for Clocks.—The use of the pendulum in clocks may be explained by Fig. 16. The pendulum swings between two arms *a*, and is connected with the rod *o* and the *escapement mn*. The *pallets* of the escapement work into the teeth of the *escapement-wheel R*. When the pendulum swings, one of the teeth of the wheel escapes from the pallet *m*, and the clock weight (or spring) which acts through the train of wheels falls a little and moves *R* forward. But no sooner has *m* released a tooth than *n* catches another, which cannot be released till the pendulum swings back and lifts *n* out. So the teeth are released just as rapidly as the pendulum swings. Heat and cold lengthen and shorten ordinary pendulum-rods, and thus change the rate of the clock. Many pendulums have mechanical devices by which the heat, while it lengthens the rod. raises the weight, and *vice versa*, and thus keeps the pendulum of the same length. The hands and face of a clock are simply to keep count of the number of times the pendulum vibrates, and thus save us very monotonous work!

MACHINES.

101. **The Mechanical Powers.**—All machines, however complex, are combinations of one or more of the six *mechanical powers.*—viz., the *lever*, *wheel and axle*, *pulley*, *inclined plane*, *wedge*, and *screw.*

These six mechanical powers, and the many combinations of them in machinery, are intended, in a general sense, to increase either the efficiency of the power applied to the machine, or the rate of motion.

102. **Law of Machines.**—It is a universal law in mechanics that *power is gained only at the expense of speed, and speed is gained only at the expense of power.*

This follows from our definition of momentum. A heavy weight moving slowly has as much moving force as a light weight moving rapidly. In the mechanical powers we always consider a *weight* or *load* to be moved (or resistance to be overcome) and a *power* to do the work. In calculating the efficiency of a machine, the power and load are considered to be balanced, or in equilibrium. If the power *moves* the load, it must be greater than the amount calculated for equilibrium, and enough greater at least to overcome the friction of the machine and the inertia of the load.

THE LEVER.

103. **The Lever.**—A lever is any bar or rod which is used to pry or move a weight, by having the power applied to one point which is free to move, the weight supported on another movable part, and another point resting on or against an immovable support. This support is called the fulcrum, and there are three classes of levers, depending upon the position of the fulcrum.

In levers of the first class the fulcrum is between the power and the weight. (Fig. 17.)

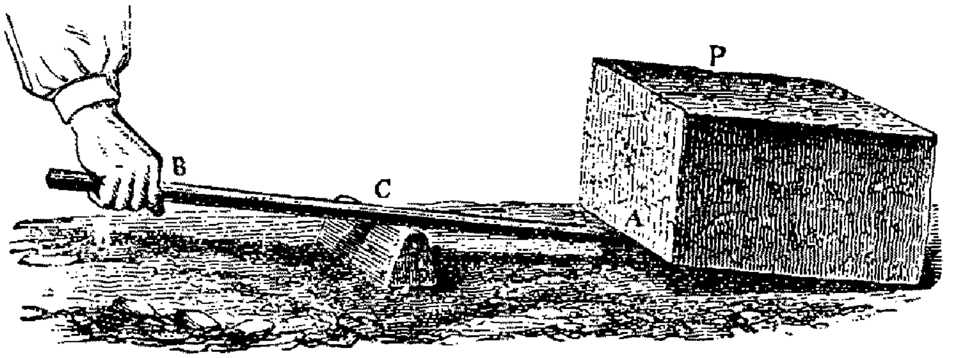

FIG. 17.—LEVER OF THE FIRST CLASS.

In levers of the second class the weight is between the power and the fulcrum. (Fig. 18.)

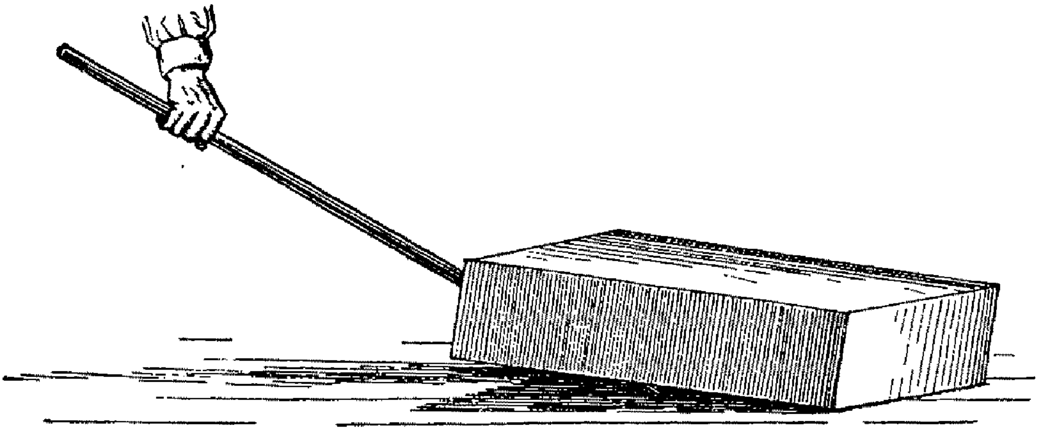

FIG. 18.—LEVER OF THE SECOND CLASS.

In levers of the third class the power is between the weight and the fulcrum. (Fig. 19.)

Questions.—What kind of lever is a balance? a see-saw? a pair of scissors? a ladder raised by a man near its base? the forearm of a man? a pair of tongs? pincers? a wheelbarrow? sheep-shears? the handle of a water-pump? a claw-hammer used in drawing a nail? the rudder of a ship?
Where is the fulcrum in each case?

104. **Lever-Arms and Leverage**.—The part of any lever between the power and the fulcrum is called the *power-arm*

of the lever, and the part between the weight and the fulcrum is called the *weight-arm*. *The ratio of the power-arm to the weight-arm is called the leverage, and represents how many times greater the weight is than the power*, or the number of times the power is multiplied by using the lever.

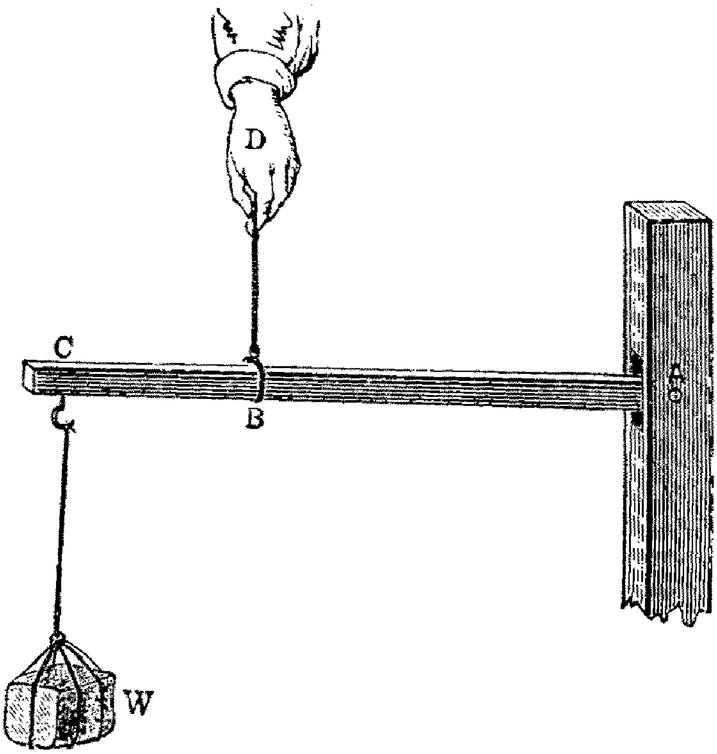

FIG. 19.—LEVER OF THE THIRD CLASS.

105. Law of the Lever.—The italics in the last article give one expression of the law of the lever. That law may be expressed in many ways. In proportion it would be

$$p : w :: \text{weight-arm} : \text{power-arm}.$$

As an equation this becomes

$$p \times \text{power-arm} = w \times \text{weight-arm},$$

or, the moment of the power = the moment of the weight.

Note.—In the above, p represents the power, and w the weight, and these letters will be so used hereafter in all problems relating to machines.

106. In levers of the second class the power-arm is the whole lever, and therefore always greater than the weight-

arm. In levers of the third class the weight-arm is the whole lever, and always greater than the power-arm. In levers of the first class, each arm is a part, only, of the lever, and the position of the fulcrum determines which is longer. Therefore power is always gained by using a second-class lever, always lost by using a third-class lever, and gained or lost by using a first-class lever, depending on whether power or weight has the longer part of the lever.

107. To gain intensity of force by the use of a lever we want all the *leverage* possible, and this is accomplished by getting the weight as near as possible to the fulcrum. Notice that the spaces traversed by power and weight are inversely as the power is to the weight. (Art. 102.) Any force or weight multiplied by the distance it moves is called the "work done" by that force or weight. Hence the law applied to all machines:

The work done by the power is equal to the work done by the weight or load.

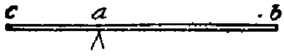

FIG. 20.

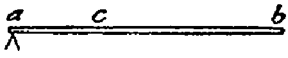

FIG. 21.

FIG. 22.

Exercises.—(In these problems, the power is always applied at b, the weight rests at c, and a is the fulcrum.)

1. In Fig. 20, $ab = 10$ inches, $ac = 2$ inches, $p = 20$ pounds: find w.
2. In Fig. 20, $bc = 22$ inches, $p = 40$, $w = 400$: find ac.
3. In Fig. 20, $ab = 12$ inches, $ac = 3$ inches, $w = 40$ pounds: find p.
4. In Fig. 20, $p = 16$, $w = 240$: find the *leverage*.
5. In Fig. 21, $ab = 20$, $ac = 4$, $w = 75$ pounds: required p.
6. In Fig 21, $ab = 24$ inches, $p = 100$ pounds, $w = 300$ pounds: required bc.
7. In Fig. 22, $ab = 1$ inch, $ac = 12$ inches, $w = 20$ pounds: required p.
8. In Fig. 22, $ab = 2$ inches, $ac = 8$ inches, b moves at the rate of 3 feet per second, at what rate does c move? $w = 3$ pounds: required p.

Experiment 19.—Take a solid foot-rule, or better, a metre-stick, marked to centimetres. Procure two empty tomato-cans with wire handles, and a few pounds of nails. Place the metre on a proper fulcrum, and, with a can suspended from each end, balance it quite accurately with the nails. Then count any number of nails into one can, and verify the law of the lever by counting the number required in the other can to balance it. Use a spring-balance for power, with a lever of the third class.

108. **The Balance.**—The balance is a lever of the first kind. Its accuracy will depend on the exact equality of the

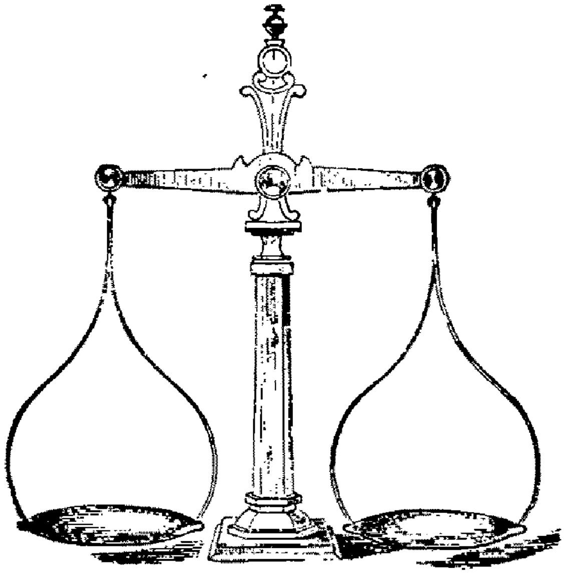

FIG. 23.—THE BALANCE.

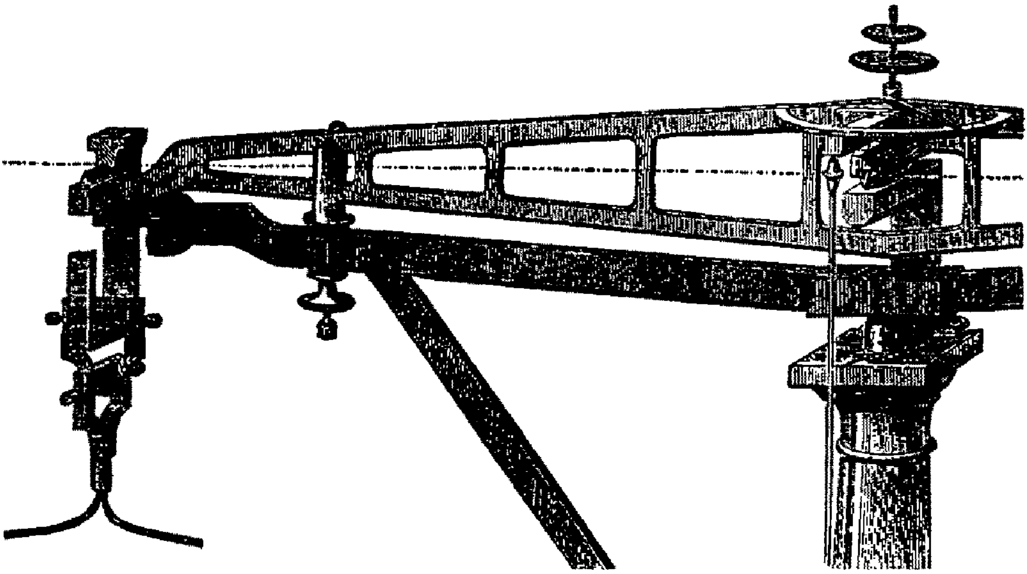

FIG. 24.—ARM OF A DELICATE BALANCE.

two arms, and may be tested by first weighing a substance, then reversing weights and substance. If they still balance, it is correct.

THE WHEEL AND AXLE.

109. Definition.—The wheel and axle as a mechanical power consists of a wheel attached to an axle so that they turn together. The power is applied to the circumference of the wheel, and the weight is hoisted by a cord wound on the axle. A crank may take the place of the wheel.

110. Law of Wheel and Axle.—The principle of the wheel and axle is the same as that of the lever.

The radius of the wheel ab is the lever-arm of the power, and the radius of the axle ac is the lever-arm of the weight. There is equilibrium when

$$p \times ab = w \times ac.$$

Having given any three of these, the fourth can be found as in the case of the lever.

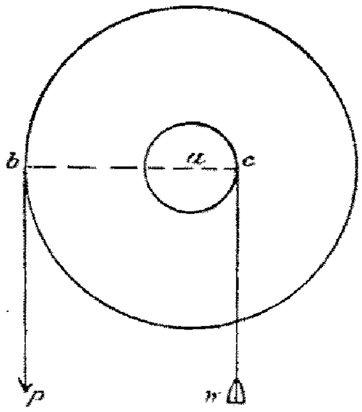

FIG. 25.—WHEEL AND AXLE.

The other statements of the law of the lever apply also to the wheel and axle.

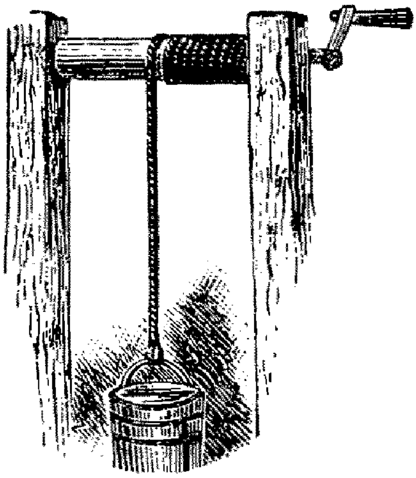
FIG. 26.—WINDLASS.

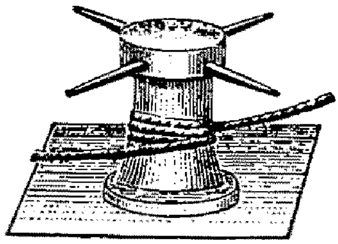
FIG. 27.—CAPSTAN.

The windlass (Fig. 26) and capstan (Fig. 27) are examples of the wheel and axle.

111. **Cog-Wheels.**—If the wheel or the axle has teeth which work into similar teeth in other wheels, we will have a train of cog-wheels. The law of equilibrium of such a train is: the weight multiplied by the product of all the radii of the axles is equal to the power multiplied by the product of all the radii of the wheels.

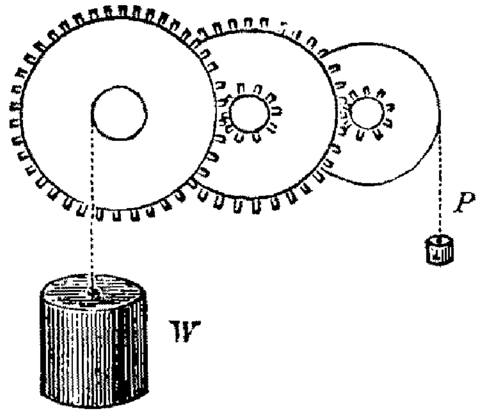

FIG. 28.—TRAIN OF WHEELS.

Since the teeth of a small wheel are the same distance from one another as the teeth of a larger wheel in which it works, when it makes a complete revolution the larger one has only turned part way round. If one has half as many teeth as the other, it will make two revolutions to one of the other. It will, therefore, travel twice as fast. But the number of teeth is proportional to the circumferences, and hence to the radii, of the wheels. Hence we have the principle that the velocity of connected wheels is inversely proportional to their radii.

112. **Train of Wheels.**—An axle with cogs is called a *pinion*. If a power turns a wheel the pinion of which works in another wheel, the pinion of this in another wheel, and so on, we have great increase of power, but we lose velocity. If we apply our power to the other end of the train, the last wheel, we gain great velocity when we reach the first pinion, but we lose power in the same proportion. The first method is used when we want a small power to move a heavy weight, and the latter when we want to gain a great velocity.

Wheels may also be connected by means of belts. The circumstances of motion are the same as in a train of cog-wheels. In this case the friction between the belt and the surface of the wheel takes the place of the cogs, and the advantage is that power can be communicated through a long distance.

Exercises.—In the following examples let R stand for the radius of the wheel and r for the radius of the axle.

1. Given $R = 20$, $r = 5$, and $P = 200$, to find W.
2. Given $R = 20$, $P = 100$, and $W = 1000$, to find r.
3. Given $R = 20$, $r = \frac{1}{4}$, and $W = 500$, to find P.
4. Given $r = \frac{1}{2}$, $W = 1000$, and $P = 40$, to find R.
5. In lifting an anchor which weighs 1000 pounds, four men work a capstan having a radius of 2 feet, by bars the outer ends of which are

6 feet from the centre of the barrel. How much force does each exert? *Ans.* 83.3 + pounds.

6. A power of 5 pounds acts on a wheel with a radius of 1 foot. The pinion (2 inches radius) acts in a wheel of 1 foot radius. This is repeated 3 times. What weight may be lifted? *Ans.* 1080 pounds.

7. Given $R = 50$ centimetres, $r = 10$ centimetres. The power, 10 kilograms, required W. The power falls 20 metres. How far will it hoist W?

THE PULLEY.

113. **Fixed Pulley.**—The pulley consists of a wheel working in a block. In its simplest form it is used to change the direction of a force. In this case there is no power gained; a little is lost by friction and by the stiffness of the rope; but, except these, it is carried over without loss or gain. In Fig. 29 the downward force becomes an upward one, and can be applied to lifting weights. Such a pulley is called a *fixed pulley.*

114. **Movable Pulley.**—The case is different when we have a pulley such as is shown in Fig. 30. Here the

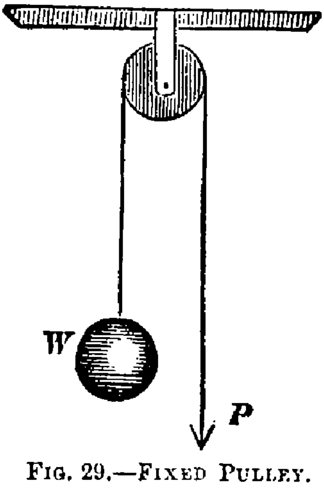

FIG. 29.—FIXED PULLEY.

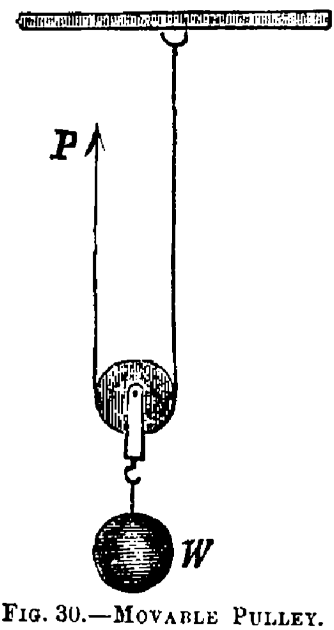

FIG. 30.—MOVABLE PULLEY.

weight is supported by both branches of the cord above the pulley, hence the tension on each need be but half the weight; that is, for equilibrium, W must be twice P.

A pulley of this kind will, therefore, enable a power of one pound to lift a weight of two pounds. Such a pulley is called a *movable pulley.*

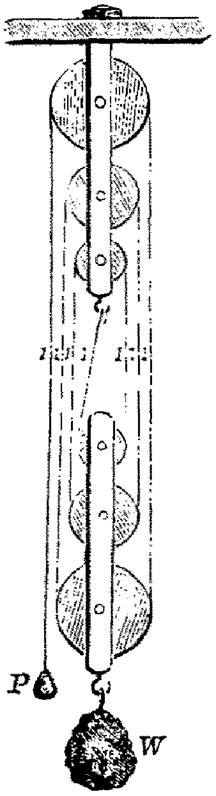

Fig. 31.—Combination of Pulleys.

115. **Work done.**—Since, when W is lifted any distance, the pulley is elevated the same amount, the ropes at both a and b will be shortened, and P will have to rise through twice this distance. Hence, as in the lever, in order to gain the advantage of the movable pulley, we lose space and time. The work done by the power is equal to the work done by the weight. (Art. 107.) While in motion, the momentum of the power is equal to the momentum of the weight.

116. **Combination of Pulleys.** — Fig. 31 represents the theory of movable pulleys. To estimate the power gained let us suppose a force of one pound applied at P. Disregarding friction, this force of one pound is felt throughout the whole length of the rope, and, as the rope pases six times to the movable block, it (the block) will be supported by a force of six pounds.

117. **Law of the Pulley.**—*In a combination of pulleys with one rope, a power will balance a weight as many times greater than itself as the number of times the rope passes to the movable block.*

118. **The Differential Pulley-Block.**—Fig. 32 represents the differential pulley, which may be made to gain power to any extent, and is a complete example of the principle of mechanics stated in Art. 102.

The fixed pulley, or "differential pulley-block," consists of two wheels, or "sheaves," cast solidly together. The circumference of each of these sheaves is pocketed to carry the links of the chain and prevent slipping. The sheave at the back, or right-hand side of the block, contains one more pocket, at least, than the front sheave. To hoist the load, the operator pulls the chain at *P*. When a given number of links, say 40, have passed through his hand, the wheel *A* has made one revolution. This takes up 40 links at *B*, and lets out 38 links at *C*, which shortens the double chain *BC* 2 links and raises *W* *one* link. (Each pocket counts *two* links.) As the operator pulls *P* forty times as far as *W* rises, 1 pound will balance 40 pounds. This pulley will sustain its load at any height without running down, by the friction of the chain and bearings. This is a great advantage. To lower *W*, the operator pulls the chain at *L*.

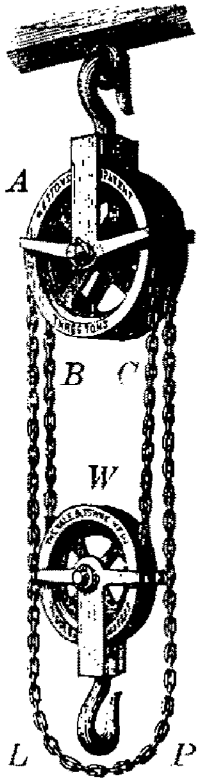

FIG. 32.—DIFFERENTIAL PULLEY.

The wheel *A* may be made of any size, thus diminishing the ratio of the two sheaves and increasing the power accordingly. Mechanical devices may be used to turn *A*, thus further increasing the lifting power of the pulley.

Exercises.—1. In Fig. 33, how much weight will a power of 20 pounds lift?

2. If the power moves through 30 feet, how far will the weight move?

3. How much power will be required to lift a weight of 1 kilogram through 1 metre, and through what distance will it move?

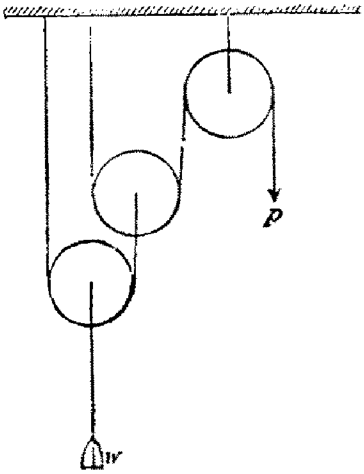

FIG. 33.—PULLEYS.

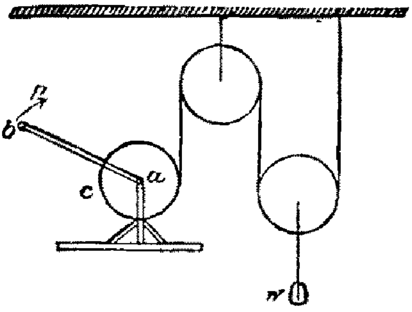

FIG. 34.—PULLEY AND WINDLASS.

4. In a system of pulleys with one rope a power of 2 pounds balances a weight of 24 pounds: how many movable pulleys are employed?

5. In the combination of pulley and windlass of Fig. 34, *ab* is 2 feet, *ac* 6 inches. A power of 30 pounds is applied at *b*: how much weight can be lifted?

6. How many turns will be required to lift the weight through 3 feet?

7. In a differential pulley-block there are 31 and 30 pockets respectively on the two sheaves. $P = 10$ pounds: required W.

8. In the differential pulley shown in Fig. 32 there are 20 and 19 pockets in the upper wheel. $W = 1000$ pounds, and the friction adds 2000 pounds: required P.

THE INCLINED PLANE.

119. **Definition.**—The inclined plane is any plane surface inclined to the horizon. A board for rolling barrels into a wagon, a railway track not level, a coasting hill, are inclined planes.

120. We best illustrate the use of the inclined plane by the board, or "skids," for loading a wagon with barrels. The skids are used to avoid the necessity of lifting the barrel perpendicularly. The work done is the weight of the barrel multiplied by the height of the wagon. A boy who could not at all lift the barrel, could roll it up the skids. In the railway, or in the common road, we may not wish particularly to place the load on the hill-tops, but if a hill-top is in the line of the road, and cannot be cut away, we must surmount it to gain the farther side, and the load must be raised the height of the hill. The slope of a railway is called its *grade*, and the steepness of the grade is generally denoted by giving the number of perpendicular feet per mile. In a road with a grade of 66 feet per mile, the rise would be 66 feet in 5280, or 1 foot in 80. For a small plane we would call this a grade of 1 in 80. On such a plane a pressure of 1 pound would hold a weight of 80 pounds; that is, would prevent it from rolling down, the other 79 pounds being supported by the plane. This holds good in any case, and gives us the

121. **Law of the Inclined Plane.**—*A given power, acting parallel with the plane, will balance a load as many times greater than itself as the length is times greater than the height of the plane.*

Here the work done by the power is the power multiplied by *the whole distance it moves*, and the work done by the weight is the weight multiplied by the distance it *rises vertically.*

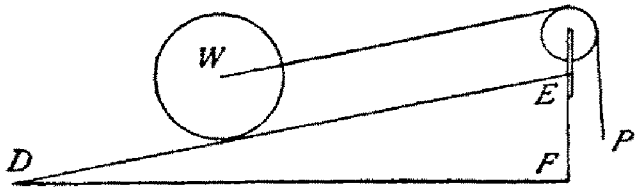

FIG. 35.—INCLINED PLANE.

Exercises.—1. In Fig. 35, $DF = 20$, $EF = 4$, $W = 500$ pounds: find P.

2. In the same figure with the same dimensions, $P = 250$ pounds: find W.

3. A boy, who can push 100 pounds, wishes to roll a barrel of oil, weighing 500 pounds, into a wagon 4 feet high: how *long* must his skids be?

4. A locomotive which can exert a continuous pull of 15,000 pounds on a train is required to draw a train weighing 990,000 pounds: what is the steepest grade it can overcome?

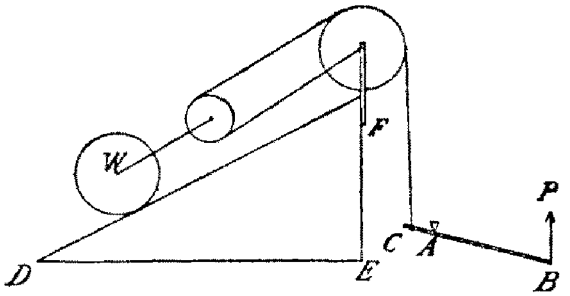

FIG. 36.—COMBINATION OF POWERS.

5. In the combination of lever, inclined plane, and pulley of Fig. 36, $AB = 10$ feet, $AC = 2$ feet, $DF = 20$ feet, $EF = 8$ feet, $P = 100$ pounds: how large a weight can be lifted?

6. How much power will be needed to lift a ton?

7. How far will P have to move to drag W through 1 foot?

THE WEDGE AND SCREW.

122. The Wedge.—If the inclined plane is pushed under the body, it becomes a wedge, and the same rules for equilibrium hold good. The height of the plane is now the back of the wedge, and the weight is as many times greater than the power as the length exceeds the back of the wedge.

Wedges are used for splitting timber, for raising heavy weights, for cutting and piercing. Knives, scissors, awls, chisels, pins, needles, are wedges.

123. **The Screw.**—A screw is an inclined plane wound around a cylinder.

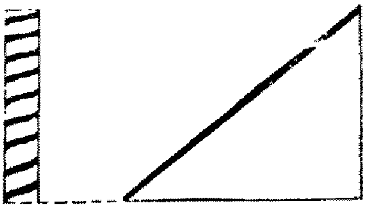

Fig. 37.—Screw and Inclined Plane.

Experiment 20.—Take a triangle of paper, as in Fig. 37, and wind it around a cylinder of wood; it will illustrate how an inclined plane can be made into a screw.[1]

124. **Law of the Screw.**—One complete turn of the screw will lift the weight through the distance which separates the threads. The law of the screw is, therefore, that the pressure exerted is as many times greater than the power as the circumference described by the power is greater than the distance between the threads.

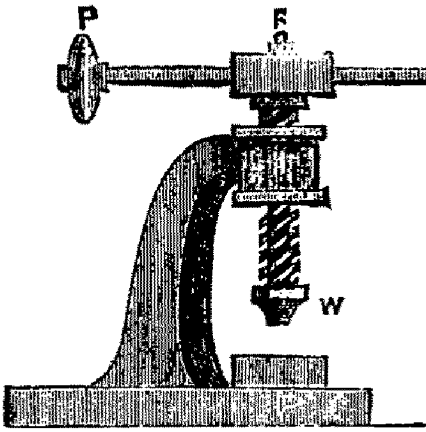

Fig. 38.—The Screw.

Exercise.—A power of 30 pounds applied at the end of a lever 2 feet long acts on a screw, the distance between the threads of which is $\frac{1}{20}$ of an inch: how much weight can be lifted?

In the common screw, propelled by a screw-driver, the weight is the resistance of the material penetrated, and the circumference described by the power is the circle through which the largest part of the handle travels.[2]

125. **Friction.**—All the laws of machines are modified by *friction.* Friction is roughness at the point of contact of two surfaces, which prevents them from sliding freely on each other. In levers there is friction at the fulcrum, in the wheel and axle and pulley at the bearings, on the

[1] Such a curve is a helix, and not a spiral, as often stated. A spiral is a curve in one plane.

[2] The distance between the threads of a fine screw is best obtained by measuring an inch along it and counting the number of threads.

inclined plane, wedge, and screw, at their surfaces. In all these cases this represents so much resistance, to overcome which additional power is required. It is important to ascertain the amount of friction between surfaces of different kinds, so that its effect may be accurately taken into account in our theories of machines. The following will afford a means of testing its amount.

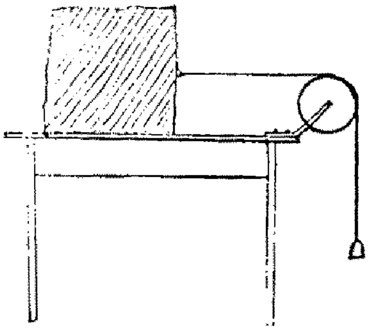

FIG. 39.—DETERMINING FRICTION.

Experiment 21.—Fasten a pulley to the table, as in Fig. 39. Place a block on the table and attach the pulley-cord to it. On the other end of the cord apply weights till the block begins to move. The amount of these weights will measure the friction between the block and the table.

Experiment 22.—Place a brick on end, then on face on the table; the friction will be the same in both cases.

Place a second brick on top of the first; the friction will be doubled.

126. **Laws of Friction.**—By some such arrangement as this it has been found,—

1. That friction is less between metals of different kinds than between metals of the same kind. Hence the advantage of brass bearings for iron axles.

2. That it is proportional to the weight (or pressure), and does not depend on extent of surface in contact.

3. That it is greater at the start than after motion has commenced. A part of the weight may be removed from the cord, and it will continue to descend.

The object of lubricants is to diminish friction.

127. **Friction Essential.**—Friction should not be looked upon as a resistance merely: it is indispensable to our welfare. It is the friction between our feet and the ground which saves us from falling at every step. It is the friction between the particles of dirt and the rocks which prevents all the hills from crumbling down and everything being reduced to a dead level. It is the friction of nails and screws which gives them their utility and prevents all our struc-

tures from falling to ruins. It enables the engine to draw us on the track; it gives to belted wheels their value; it enables us to make long ropes of short strands, and keeps knots tied; it prevents the rivers from flowing with the velocity of falling bodies.—(Art. 156.)

128. **Machines do not create Energy.**—We have seen both in the lever and in the pulley that the work done by the power is equal to the work done by or upon the weight or resistance. This is a general law of machines. Whenever we gain power we lose speed, and when we gain speed we lose power. A machine cannot create any energy. It transmits that which is applied to it by an external power. The power does work upon it, and it does work upon the resistance. This work may be of a different kind, but is the same in amount.

129. **Uses of Machines.**—The question then comes up, What do we gain by machines? Sometimes we gain only a change of direction, as in the fixed pulley; sometimes it is an advantage to gain power at the expense of velocity, as in a lever or pulley used to raise a heavy weight; and sometimes it is an advantage to gain velocity at the expense of power, as in the case of a clock, where the slow falling of the weight, or uncoiling of the spring, may cause more rapid motion of the hands. Sometimes it is a gain to change the character of the power, as in the steam-engine, where heat produces mechanical motion, or in electric-lighting machines, where heat and motion produce electricity and light. Machines are also a great gain in enabling us to use the power of the wind, of steam, of falling water, and of animals.

130. **Perpetual Motion.**—These examples will show the character of the gains of machinery. In no case is the energy increased by the machine itself. We see, then, the folly of all *perpetual-motion* machines,—machines which will keep themselves running without the addition of any external energy. Any such machine would have to create

energy. Let us suppose that water falling on a wheel would cause such motion of the wheel as would, applied to a pump, force the water up to the level from which it fell. This would be a perpetual-motion machine, for it would keep itself going forever without any new supplies of force. But it requires just as much energy to lift the water up to its level as is given out by the fall. But part of the energy of the fall is required to overcome the friction of the machinery and the resistance of the air, hence there cannot be enough left to raise the water to its old level. If machines could be constructed so as to run without any resistance, perpetual motion would be possible, and under no other circumstances.

Such a machine would be useless for any practical purposes, for if any machinery were connected with it, it would soon bring it to rest, and a new supply of power would be needed.

General Exercises.[1]—1. The minute-hand of a watch is twice as long as the second-hand: show that the end of the second-hand moves thirty times as fast as the end of the minute-hand.

2. Find the space described in the fifth second by a falling body.

3. If a body falls for a quarter of a minute, show that at the end of that time it would be moving at the rate of 483 feet per second, and ascertain what this velocity will be, expressed in miles per hour.

4. A stone dropped into a well is heard to strike the water in two seconds and a half: find the depth of the well. *Ans.* 100 feet.

5. An express train, 66 yards long, moving at the rate of 40 miles an hour, meets a slow train, 110 yards long, moving at the rate of 20 miles an hour: find how long a man in the express train takes to pass the slow train, and how long the express train takes in completely passing the slow train. *Ans.* $\frac{1}{15}$ minute. $\frac{1}{10}$ minute.

6. A river, one mile broad, is running downward at the rate of 4 miles an hour; a steamer can go up the river at the rate of 6 miles per hour: find at what rate it can go down the river. *Ans.* 14.

7. A moving body is observed to increase its velocity by a velocity of 8 feet per second in every second: find how far the body would move from rest in 5 seconds. *Ans.* 100 feet.

[1] In these and other exercises at the ends of the chapters a great variety is given. The teacher should make a selection adapted to the class. Many classes had better omit all of them, while some would be benefited by working them all.

8. Show that a cylinder, if placed on its flat end, will be in stable equilibrium, but, if placed on its curved surface, in neutral equilibrium.

9. A triangular board is hung by a string attached to one corner: find what point in the opposite side will be in a line with the string.

10. Find where the fulcrum must be placed that 2 pounds and 8 pounds may balance at the extremities of a lever 5 feet long.

11. The arms of a lever are respectively 15 and 16 inches: find what weight at the end of the short arm will balance 30 pounds at the end of the long arm, and what weight at the end of the long arm will balance 30 pounds at the end of the short arm.

12. A straight lever, 6 feet long, and heavier towards one end than the other, is found to balance on a fulcrum 2 feet from the heavier end, but when placed on a fulcrum at the middle it requires a weight of 3 pounds hung at the lighter end to keep it horizontal: find the weight of the lever. *Ans.* 9 lbs.

13. Two men, A and B, carry a weight of 200 pounds on a pole between them; the men are 5 feet apart, and the weight is at a distance of 2 feet from A: find the weight which each man has to bear.

14. Suppose that a body which really weighs 1 pound appears in a balance to weigh 1 pound 1 ounce: find the proportion of the length of the arms.

15. A substance is weighed from both arms of a false balance, and its apparent weights are 9 and 4 pounds: find the true weight.

16. The radius of the axle of a capstan is 1 foot: if four men push each with a force of 100 pounds on spokes 5 feet long, show that on the whole a tension of 2000 pounds can be produced on the rope which passes around the axle.

17. A wheel and axle is used to raise a bucket from a well; the circumference of the wheel is 60 inches, and while the wheel makes three revolutions the bucket, which weighs 30 pounds, rises 1 foot: find the smallest force which can turn the wheel. *Ans.* 2 lbs.

18. Suppose the power to act parallel to the plane, and that the height of the plane is to its base as 5 is to 12: if the weight is 65 pounds, find the power. *Ans.* 25 lbs.

19. Find the relation between the power and the weight in a screw which has 10 threads to an inch, and is moved by a power acting at right angles to an arm at the distance of 1 foot from the centre.

20. A pendulum vibrates 65 times in a minute: how much must it be lengthened to vibrate once in a second?

Suggestion.—Time of one vibration $= \frac{12}{13}$ seconds. *Ans.* $\frac{25}{144}$ of its length.

21. In what time would a seconds pendulum vibrate at a height of 4000 miles above the earth's surface? at a depth of 2000 miles under ground?

22. How long is a pendulum which vibrates 40 times a minute, a seconds pendulum being 39.1 inches long?

23. A seconds pendulum carried up a mountain vibrates 58 times a minute: what is the force of gravity? *Ans.* $\frac{841}{900}$ of gravity at the surface. This would be expressed by saying it would cause an acceleration in a falling body of 30.1 feet per second. Work out this result.

SUMMARY OF CHAPTER II.

Change from a state of rest to a state of motion, or *vice versa*, is always produced by the application of some *force*.

Forces are either impulsive or continuous.

An impulsive force tends to produce uniform motion, and a continuous force tends to produce accelerated motion.

The fundamental relations of motion and force are expressed in the three laws of motion first stated by Sir Isaac Newton.

The momentum of a moving body is its ability to impart motion to another body.

Forces may be measured either by the pressure which they cause or by the acceleration they can produce in bodies free to move.

The C. G. S. unit of accelerating force is the dyne. This is the force which will accelerate a mass of one gram at the rate of one centimetre per second.

The dyne in the middle United States is about $\frac{1}{980}$ of a gram.

Two forces acting on a body in any direction, not in the same straight line, will move it to the opposite corner of a parallelogram, of which the two sides represent the magnitude and direction of the two forces.

When a body in motion is acted on by a continuous force towards any point out of the line of motion, its path is a *curve*.

Work consists in *causing motion against resistance*.

The practical units of work are the foot-pound and the kilogrammetre. The invariable unit is the erg.

Power is estimated in units of *work* and *time*.

The practical unit of power is the horse-power,—*i.e.*, the ability to lift thirty-three thousand pounds one foot in one minute.

Energy is ability to do work.

Potential energy is the energy possessed by a body or a substance on account of its position, or state of compression or strain.

Kinetic energy is the energy which a body possesses on account of its *motion*. Its amount is determined by the *weight* as well as the *velocity* of the body.

A body starting to move is converting potential energy into kinetic energy; a body stopping in a position of increased advantage is converting kinetic energy into potential.

Energy is indestructible. It may undergo countless transformations, but the total amount in the universe is always the same.

The centre of gravity of a body is the point on which the body will balance.

A body is stable when, the centre of gravity being moved, it tends to return to its original position.

The earth's attraction (gravity) causes a velocity of about 32.2 feet, or 9.8 metres, per second in a body free to fall. This acceleration is practically the same for all bodies, light and heavy, and whether they have a motion horizontally or not.

Gravity *takes from* the velocity of a body moving upward 32.2 feet each second.

The resistance of the air seriously modifies the results of calculation for very rapidly-moving bodies or bodies of light material.

A swinging pendulum is a complete example of the conversion of potential into kinetic energy, and *vice versa.*

All machines are combinations of the six mechanical powers, the lever, wheel and axle, pulley, inclined plane, wedge, and screw, some of them frequently much modified.

In all machines velocity of motion is gained only at the expense of intensity of pressure, and *vice versa.*

The principle of all machines is that the *power* multipled by the distance it *moves* is equal to the *weight* multiplied by the distance it *rises.*

All the laws of machines are much modified in practice by rigidity of ropes and other material, and by various other resistances, the principal one of which is *friction.*

No machine, however nearly perfect, can create or originate any energy, power, force, or work: it simply affords a convenient method of applying the kinetic energy of moving air and falling water, the radiant energy of light and heat, the subtle energy of electricity, and the vital muscular energy of ourselves and our domestic animals, to such purposes as we choose.